ESSAYS ON DYNAMIC MACROECONOMICS

by

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ABSTRACT

This thesis consists of four independent papers in macroeconomics.

Labor Supply and Consumption under Uncertainty: Prudence Reconsidered examines how variations in labor supply can be used to self insure against individual specific risks. It is found that precautionary saving can be significantly underestimated if wages fluctuate and labor supply is assumed fixed when it is not. Moreover, the standard definition of prudence is shown not to be applicable in the current setting, and a new definition of prudence is suggested.

Idiosyncratic Risk in the U.S. and Sweden: Is there a Role for Government Insurance? examines the welfare effects of redistributive taxation when agents are subject to uninsurable income risk. To calibrate the model, wage processes are estimated on Swedish and U.S. data. The estimation results show that there is more individual specific wage risk in the U.S than in Sweden. Although the model predicts that distortions are significant, the welfare benefits of government redistribution and insurance systems can be substantial.

The Effectiveness of Government Debt and Transfers as Insurance takes the previous paper one step further by also allowing for government debt as a policy instrument. It is found that both debt and transfers can help households to smooth consumption over time, but that debt also has negative effects on equity. When used in isolation, debt will enhance welfare if transfers are lower than optimal. However, public debt has no positive effects if transfers are used optimally.

Endogenous Monetary Policy and the Business Cycle makes the central bank's monetary policy decisions endogenous in a business cycle model by letting the central bank set money supply to minimize the volatility of inflation and output. It is found that small changes in the central bank's preferences can generate large

changes in the derived money supply rule and in the business cycle behavior of nominal variables, but changes in the money supply rule do not generate any major changes in the behavior of real variables.

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Chapter 1

Introduction

The broad and somewhat vague title of this thesis suggests that the four papers it contains are linked more by method and approach than by topics. This is true although the first three papers are also related in contents. 'Dynamic' in the title is intended to convey a broader message than the literal meaning of the word. I do believe that intertemporal considerations are important, but even more important is the concomitant equilibrium approach to economic modeling. All problems are explicitly stated at the individual and firm level, and agents behave rationally to maximize their life-time utility.

This micro founded equilibrium approach is sometimes mistakenly thought to imply assumptions of frictionless and complete markets. On the contrary, a common theme in the present papers is assumptions of missing insurance markets, borrowing constraints, and sticky wages. My view is that the models belong in the dynamic equilibrium framework as long as the deviations from perfect markets are clearly stated and as long as the rules under which agents are allowed to act are explicitly formulated.

When future income is uncertain, and when people cannot insure against this uncertainty, they will typically try to shift resources into the future and build up a buffer stock of savings. This motive for precautionary savings is fairly well explored in the standard setting where agents derive utility from a single consumption good. Kimball (1990) has derived a measure of prudence which is useful for classifying utility functions according to their associated tendency for precautionary saving. The problem agents face in Chapters 3 and 4 is slightly different. Future wages are uncertain, and agents can choose both consumption and labor supply in each period. When working with these models I started to consider what happens to precautionary savings when labor supply is a choice variable.

This resulted in **Chapter 2**, where I explore the following questions: Will people use variations in labor supply to self insure against income risk? If so, is there then less incentives to engage in precautionary savings? And is it possible to rank utility functions according to the implied precautionary savings as Kimball did?

I find, maybe surprisingly, that there is often more precautionary saving when labor supply is flexible than when it is fixed. Part of the intuition is that when agents are subject to wage risk, unfavorable outcomes are associated with low wages and inexpensive leisure. Increasing labor supply in response to negative wage shocks is then an inefficient insurance strategy. Rather, people choose to work harder and increase savings in good times.

I also find that the precautionary effects on leisure are similar to those on consumption. People work more today and less tomorrow if risk increases. Moreover, I show that the standard definition of prudence, as proposed by Kimball, is not applicable in the current setting and I suggest a new definition.

In **Chapter 3**, written with Jesper Lindé, we examine the welfare effects of redistributive taxation. Varian (1980) and Eaton and Rosen (1980) demonstrated how progressive tax schedules can work as implicit insurance and improve welfare if agents are unable to insure on private markets. A redistributive tax system will improve the worst possible outcome relative to the best possible outcome, but in addition to the positive insurance effect, taxes distort labor supply and savings decisions. We allow for these trade-offs and quantify the social welfare associated with different levels of redistribution.

To calibrate the model we need to find wage processes that capture the uncertainty faced by individuals. We do this by using U.S. and Swedish micro data on labor supply and earnings. The estimation results show that wages in the U.S. are more persistent and somewhat more volatile on a year-to-year basis than wages in Sweden. In addition, permanent wage differences between individuals (due to e.g. different ability or education) are considerably more pronounced in the U.S. than in Sweden.

When the model is calibrated with these two wage processes and solved numerically, we find that welfare benefits of government redistribution and insurance systems can be substantial although distortions from increased taxes are significant. In the U.S., where the wage process is most volatile, the optimal transfer level is 18 percent of output, while the optimal transfer level is two percent of output in Sweden.

In **Chapter 4**, the analysis of Chapter 3 is extended by allowing for a second policy instrument, government debt. Government debt helps agents to self insure by increasing the liquidity in the economy. If physical capital is held fixed, an issue of government debt increases the amount of financial assets but the interest rate is unaffected. On the other hand, if people want to increase their saving and government debt is held fixed, the stock of physical capital must increase and the interest rate must fall. Woodford (1990) demonstrated these effects analytically while Aiyagari and McGrattan (1998) solved a dynamic equilibrium model numerically and quantified welfare effects associated with different levels of debt.

I find that both debt and transfers can improve insurance possibilities significantly, but that debt also has negative effects on equity. When used in isolation, debt will enhance welfare if transfers are lower than optimal. However, if enough insurance is provided from the tax and transfer system, there is no positive role for public debt. The optimal policy combination in a model calibrated for the

U.S. economy is to have transfers at 22 percent of output and to let the public debt be -75 percent of output. The welfare gain of having this policy instead of the actual policy would be 3.2 percent of annual consumption.

The last paper, **Chapter 5**, was inspired by two quite different literatures on money and monetary policy in the aggregate economy. One branch, stemming from the real business cycle literature, has tried to add money to the stochastic general equilibrium models to investigate if this results in plausible predictions of the business cycle behavior of nominal variables such as money, prices, and inflation. These models are calibrated with a money supply rule which is typically estimated on post-war U.S. data (see, for example, Cooley and Hansen [1995] and Yun [1996]). The other branch consists of empirical estimations of the effects of monetary shocks. The strategy is usually to identify and estimate innovations to money supply in vector autoregressions. Recent and influential contributions are Bernanke and Mihov (1998) and Christiano, Eichenbaum, and Evans (1996).

These papers, particularly those stemming from the real business cycle literature, spurred me to ask how important the assumption of a stable money supply rule is. What happens if preferences and objectives of the central bank change? Will the money supply rule change substantially, and if the rule changes, will then the business cycle behavior of nominal variables change? These are the questions I address in Chapter 5. I do this by letting the central bank minimize a loss function in inflation and output volatility to solve for the optimal money supply rule in a business cycle model. Gavin and Kydland (forthcoming) also examined the effects of changes in the money supply rules in a similar framework but this chapter goes one step further by connecting money supply to central bank preferences. While studies in the empirical branch have often allowed for changes in the conduct of monetary policy (and also often found evidence of such regime shifts), the studies must still rely on regimes being stable over some (not too short) time periods. My paper contributes to this literature by showing that even what appears to be minor changes in central bank preferences can have significant effects on the chosen money supply rule.

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Chapter 2

Labor Supply and Consumption under Uncertainty: Prudence Reconsidered*

1 Introduction

How can and how do people use variations in labor supply to self insure against individual specific risks? In spite of the significant amount of work which has been devoted to both theoretical and quantitative studies of precautionary saving, this question seems to be unanswered.

Pratt (1964) showed that for an agent with utility function $u(c)$, where c is consumption, $-u_{cc}/u_c$ is a good measure of his absolute degree of risk aversion. While risk aversion is a concept closely related to an agent's utility, prudence and precautionary saving relate to the agent's behavior when facing uncertainty. Leland (1968) and Sandmo (1970) first formalized this concept and showed that a positive third derivative of the utility function is crucial for obtaining positive precautionary saving. Kimball (1990) paralleled Pratt's analysis and showed that a good measure of the absolute degree of prudence is $-u_{ccc}/u_{cc}$.

In this paper, I examine the precautionary behavior of agents who can choose

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their labor supply in each period. First, I look at the concepts of risk aversion and prudence in this setting, and relate them to Kimball's study. Then, the theoretical implications of prudence are analyzed in a simple two-period model. In particular, I examine how agents use variations in labor supply to self insure against risk, and what implications this has for the magnitude of precautionary savings.

The standard measure of prudence, as defined by Kimball, is not directly applicable in this setting. The reason is that it measures precautionary strength in terms of the uncertain variable. A natural requirement of the prudence measure is that it indicates how decision variables respond to uncertainty. For example, when studying precautionary saving and wage uncertainty, we want an agent with high prudence to save more than an agent with low prudence. The standard definition of prudence works well when labor supply is fixed, since wage and endowment shocks are then equivalent. But if labor supply is flexible, prudence needs to be measured in the same terms as savings and not in terms of the wage rate. In the next section I look in to this problem and suggest an alternative definition of prudence, which will fulfill the requirement under more general conditions.

Eaton and Rosen (1980) considered a framework where agents determine labor supply before wage uncertainty is resolved. They showed that labor supply can increase in response to increased wage uncertainty if risk aversion is sufficiently high. I show here that when allowing for an interaction between labor supply and saving decisions, increased wage uncertainty unambiguously lowers future labor supply. In another related paper, Bodie, Merton, and Samuelson (1992) analyzed how labor supply flexibility influences investors' portfolio decisions. One finding was that greater labor supply flexibility induces more risk taking. Therefore, one might expect that greater labor supply flexibility also would make agents less prudent. However, this will not always be the case.

The analytical and numerical investigations in this paper show that the degree of prudence often will be underestimated if wages fluctuate and labor supply incorrectly is assumed to be fixed. Of course, flexibility is always good for the agents, so even if their precautionary savings increase, their expected utility is higher with flexible labor supply. With fixed labor supply, all effects of a negative shock must be absorbed by consumption. With flexible labor supply, hours worked can be adjusted to alleviate the effect of the shock. With the same argument we note that a certain amount of saving is less costly for agents when labor supply is flexible. Therefore, agents with flexible labor supply are willing to expose themselves to more risk but they can more easily save to self insure against the uncertainty.

This last point, that flexibility makes saving easier, resembles the Le Chatelier-Samuelson principle (see Samuelson, 1972). The contents of this principle is that the elasticity of demand of one variable is greater when other variables are allowed to adjust to price changes than when other variables are held fixed. In the present case, the amount of uncertainty is related to the value of saving. Here then, saving will increase more in response to increased uncertainty if labor supply is flexible, provided that this effect dominates the effect on risk tolerance.

The usual measure of prudence is closely related to risk aversion. Kimball and Weil (1992) broke that link, and showed that both high risk aversion and high intertemporal elasticity of substitution tend to imply much prudence. The present paper illustrates this point. If an agent has decreasing absolute risk aversion, he can insure against wage fluctuations by bringing much wealth into the risky period, and a high intertemporal elasticity of substitution makes it less costly for the agent to shift wealth across periods.

Aiyagari (1994) found the precautionary savings effects to be modest in a quantitative general equilibrium model with fixed labor supply, at least for plausible parameterizations. As noted above, ignoring labor supply flexibility can make us underestimate the amount of precautionary saving. Moreover, this effect is particularly large when risk aversion is low. So, this paper indicates that Aiyagari's results must be reevaluated if we believe that labor supply is a decision variable for the individuals.

In recent work, Low (1999) examined the effects of allowing for flexible labor supply in a plausibly calibrated life-cycle economy. For most parameterizations of the model, there are more precautionary savings when labor supply is flexible. Further, as uncertainty over future wages increases, people choose to work more when young and less when old. These findings are in line with those in the present paper.

The remainder of the paper is structured as follows. In Section 2, I suggest a way of measuring risk and prudence in this framework. Thereafter, in Section 3, I use the measure of prudence to look at a two-period economy where there are endowment or wage shocks in the second period. This results in implications for the amount of precautionary savings for agents with flexible and fixed labor supply, respectively. Numerical examples show that agents with flexible labor supply often choose to hold more precautionary savings than agents who cannot choose their labor supply. In Section 4, I consider the effects that uncertainty has on labor supply. It is shown that more wage uncertainty unambiguously has a positive effect on labor supply today but a negative effect on future labor supply. Section 5 concludes.

2 Measuring risk aversion and prudence

This section starts by showing how to measure risk aversion in a framework that can allow for labor supply flexibility. This is a straightforward application of Pratt's (1964) analysis. Thereafter, an example demonstrates that the standard definition of prudence is not applicable in this setting. Finally, a new definition of prudence is suggested and contrasted to Kimball's (1990) definition.

Consider an agent who has an indirect utility function $V(a, \varepsilon)$, where a is wealth in the beginning of the period and ε is a mean zero shock during the period. In looking for a concept of risk aversion, we follow Pratt (1964) and define π as the premium the agent is prepared to pay to get $\varepsilon = 0$ with certainty, that is

$$V(a-\pi,0)=\mathrm{E}_{\varepsilon}V(a,\varepsilon),$$

where E_{ε} denotes the expectation over ε. Let $o(x)$ denote terms that vanish faster than x as x approaches zero. Denote the variance of ε by σ^2, and expand V around a and $\varepsilon=0$ to get

$$V(a-\pi,0) = V(a,0)+V_a(a,0)(-\pi)+o\left(\sigma^2\right), \tag{2.1}$$

$$\begin{aligned}\mathrm{E}_{\varepsilon}V(a,\varepsilon) &= \mathrm{E}_{\varepsilon}\left[V(a,0)+V_{\varepsilon}(a,0)\varepsilon+\frac{1}{2}V_{\varepsilon\varepsilon}(a,0)\varepsilon^2+o\left(\varepsilon^2\right)\right] \\ &= V(a,0)+\frac{1}{2}V_{\varepsilon\varepsilon}(a,0)\sigma^2+o\left(\sigma^2\right),\end{aligned} \tag{2.2}$$

where subscripts denote derivatives. We neglect terms of order $o(\sigma^2)$ and use the equality between (2.1) and (2.2), to find

$$\pi(a)=\frac{\sigma^2}{2}r(a),$$

where the degree of absolute risk aversion is defined as

$$r(a)=-\frac{V_{\varepsilon\varepsilon}(a,0)}{V_a(a,0)}. \tag{2.3}$$

While risk aversion is related to the disutility agents get from uncertainty, prudence describes agents' behavior when facing risk. Therefore, a natural requirement of the measure of prudence is that it connects the decision variable of interest to uncertainty. In most applications, we are interested in how saving reacts to risk. Consequently, we look for a measure of prudence, η, so that we can write savings as a function of η and σ^2.

Let us solve for precautionary savings in a simple two period model to demonstrate that the measure of prudence proposed by Kimball does not generally fulfill this requirement. Assume that the agent has a time-separable utility function, abstract from discounting and assume zero interest rates. Further, assume that the first and second period utility functions are identical and that the agent has no initial financial wealth. Let s be savings and ε a mean zero, second period shock, and define $\bar{v}(s)$ and $v(s,\varepsilon)$ as the indirect first and second period utility functions respectively. The agent's indirect life-time utility is then defined as

$$V(s,\varepsilon) \equiv \bar{v}(s) + v(s,\varepsilon).$$

The agent chooses savings in the first period, s, to maximize his expected indirect utility $\mathrm{E}_\varepsilon V(s,\varepsilon)$. The first order condition is

$$\bar{v}_s(s) + \mathrm{E}_\varepsilon v_s(s,\varepsilon) = 0. \tag{2.4}$$

Expand v_s and $\bar{v}_s$ around $\varepsilon = 0$ and $s = 0$, and neglect terms of $o(\sigma^2)$,

$$\begin{aligned} \mathrm{E}_\varepsilon v_s(s,\varepsilon) &= \mathrm{E}_\varepsilon\left[v_s(0,0) + v_{ss}(0,0)\,s + v_{s\varepsilon}(0,0)\,\varepsilon + \frac{1}{2}v_{s\varepsilon\varepsilon}(0,0)\,\varepsilon^2 + v_{ss\varepsilon}(0,0)\,s\varepsilon\right] \\ &= v_s(0,0) + v_{ss}(0,0)\,s + \frac{1}{2}v_{s\varepsilon\varepsilon}(0,0)\,\sigma^2, \end{aligned}$$

and

$$\bar{v}_s(s) = \bar{v}_s(0) + \bar{v}_{ss}(0)\,s.$$

Assume now that resources on average are spread equally between periods so that $s = 0$ solves (2.4) when $\sigma^2 = 0$. Then $v_s(0,0) = -\bar{v}_s(0)$, and $v_{ss}(0,0) = \bar{v}_{ss}(0)$. Hence, if (2.4) is fulfilled, we have that

$$0 = \left[-v_s(0,0) + v_{ss}(0,0)\,s\right] + \left[v_s(0,0) + v_{ss}(0,0)\,s + \frac{1}{2}v_{s\varepsilon\varepsilon}(0,0)\,\sigma^2\right].$$

We get the implied amount of precautionary saving in response to a small uncertainty by rearranging,

$$s = \frac{\eta\sigma^2}{4}. \tag{2.5}$$

where we have defined the degree of absolute prudence

$$\eta \equiv -\frac{v_{s\varepsilon\varepsilon}(0,0)}{v_{ss}(0,0)}.$$

This shows that the amount of precautionary saving is determined by the convexity of marginal utility in ε ($v_{s\varepsilon\varepsilon}$) and by the response of marginal utility to changes in the decision variable (v_{ss}). Note that this expression deviates from

Kimball's $\eta = -\frac{v_{s\varepsilon\varepsilon}}{v_{s\varepsilon}}$. However, we will see that these expressions are equivalent if labor supply is fixed or if ε is an endowment shock.

To highlight the difference between Kimball's (1990) analysis and the present, let us consider a more general framework, similar to Kimball's. Let θ denote a stochastic variable and denote its mean by $\bar{\theta}$ such that $\theta = \bar{\theta} + \tilde{\theta}$, where $\tilde{\theta}$ is the stochastic component with variance σ_θ^2. Further, let δ denote the decision variable, and let $V(\delta, \theta)$ be the agent's indirect utility function. Finally, let E_θ denote expectations over θ.

The agent chooses δ to solve

$$\max_\delta \mathrm{E}_\theta V(\delta, \theta).$$

The first order condition is then

$$\mathrm{E}_\theta V_\delta(\delta, \theta) = 0.$$

Now, Kimball measures prudence by the premium, ψ, which fulfills

$$\mathrm{E}_\theta V_\delta(\delta, \theta) = V_\delta\left(\delta, \bar{\theta} - \psi\right). \tag{2.6}$$

That is, if the agent gets $\bar{\theta} - \psi$ for sure, instead of the uncertain θ, his decision will be the same.[1] As the previous example indicates, this definition of prudence is not always relevant for the study of precautionary saving. In that example, θ is the wage rate, and δ is first period savings or consumption. We are then interested in the effects uncertainty has on the agent's decision. So, the precautionary premium we are interested in is the difference, $\hat{\psi}$, between the agent's choice under certainty and the choice under uncertainty,

$$\mathrm{E}_\theta V_\delta(\delta, \theta) = V_\delta\left(\delta - \hat{\psi}, \bar{\theta}\right). \tag{2.7}$$

Expanding these expressions around δ and $\bar{\theta}$ and neglecting terms of order $o\left(\sigma_\theta^2\right)$, we find

$$\hat{\psi}(\delta) = -\frac{V_{\delta\theta\theta}\left(\delta, \bar{\theta}\right)}{V_{\delta\delta}\left(\delta, \bar{\theta}\right)} \frac{\sigma_\theta^2}{2}.$$

[1] I only consider the "equivalent premium" here. An alternative is "compensating premia", which are additions to the uncertain variable.

Consequently, we define the degree of prudence as

$$\eta(\delta) \equiv -\frac{2V_{\delta\theta\theta}\left(\delta,\bar{\theta}\right)}{V_{\delta\delta}\left(\delta,\bar{\theta}\right)}.$$

In our previous example this amounts to

$$\eta(s) = -\frac{2V_{s\varepsilon\varepsilon}}{V_{ss}}.$$

Since $V_{ss} = 2v_{ss}$ and $V_{s\varepsilon\varepsilon} = v_{s\varepsilon\varepsilon}$ this can be rewritten as

$$\eta(s) = -\frac{v_{s\varepsilon\varepsilon}}{v_{ss}}.$$

The definition of prudence proposed in (2.7) is thus consistent with what we found in the example (cf. equation [2.5]), while Kimball's definition in (2.6) typically is not.

3 Flexibility and precautionary behavior

The first question we ask is how labor supply flexibility influences precautionary saving. The agent is assumed to have a time separable utility function, $u(c,l)$, where c is his consumption of goods, and l is his consumption of leisure. We restrict the study to utility functions which are strictly concave, i.e. $u_{cc}, u_{ll} < 0$, $u_{cc}u_{ll} - u_{cl}^2 > 0$, and 'precautionary', i.e. $u_c, u_l, u_{ccc}, u_{lll} > 0$. Further, the agent is assumed to have one unit of time to dispose of, and throughout the analysis we assume interior solutions for the leisure choice.

3.1 Endowment uncertainty

Let us first consider the effects of endowment uncertainty on the agent's behavior. Assume that first and second period wage rates are equal and non-stochastic, $w_1 = w_2 = w$. The shock ε is now a second period endowment shock, so

$$c_1 = (1 - l_1)\, w - s, \tag{2.8}$$

$$c_2 = (1 - l_2)\, w + s + \varepsilon. \tag{2.9}$$

The agent chooses consumption and labor supply to maximize

$$u(c_1, l_1) + \mathrm{E}_\varepsilon u(c_2, l_2).$$

Given the time separable utility, the leisure choice in each period is a function of contemporaneous consumption and wages,

$$u_l(c_t, l_t) = w_t u_c(c_t, l_t). \tag{2.10}$$

This, together with (5.5), (5.6), and a fixed w, defines the leisure choices as functions of savings and the shock,

$$\begin{aligned} l_1 &\equiv L_1(s), \\ l_2 &\equiv L_2(s, \varepsilon). \end{aligned}$$

The indirect first period utility function is then

$$\bar{v}(s) = u\left[(1 - L_1(s))\, w - s, L_1(s)\right],$$

and the indirect second period utility is

$$v(s, \varepsilon) = u\left[(1 - L_2(s, \varepsilon))\, w + s + \varepsilon, L_2(s, \varepsilon)\right].$$

Define

$$L_\varepsilon \equiv \frac{\partial L_2(s, \varepsilon)}{\partial \varepsilon}, \text{ and } L_s \equiv \frac{\partial L_2(s, \varepsilon)}{\partial s}.$$

Now, note that $L_\varepsilon = L_s$ and both s and ε enter additively to c_2. Hence $v_{s\varepsilon\varepsilon} = v_{sss}$. We use the Envelope theorem on the indirect second period utility function to get

$$v_s = u_c.$$

Let us restrict our analytical studies to utility functions which are additively separable in consumption and leisure so that $u_{cl} = 0$. We then get

$$\begin{aligned} v_{ss} &= (1 - L_\varepsilon w)\, u_{cc}, \\ v_{s\varepsilon\varepsilon} &= (1 - L_\varepsilon w)^2\, u_{ccc} - L_{\varepsilon\varepsilon} w u_{cc}. \end{aligned}$$

Hence

$$\eta^{\text{flex}} = -\frac{v_{s\varepsilon\varepsilon}}{v_{ss}} = -\frac{(1 - L_\varepsilon w)\, u_{ccc}}{u_{cc}} + \frac{L_{\varepsilon\varepsilon} w}{1 - L_\varepsilon w}.$$

By differentiating (2.10) we can show that $0 < L_\varepsilon w < 1$, and

$$L_{\varepsilon\varepsilon} = L_\varepsilon (1 - L_\varepsilon w)^2 \frac{u_{ccc}}{u_{cc}} - \frac{L_\varepsilon^3 u_{lll}}{w u_{cc}}$$

so that

$$\eta^{\text{flex}} = -\frac{(1 - L_\varepsilon w)^2 u_{ccc}}{u_{cc}} - \frac{L_\varepsilon^3 u_{lll}}{(1 - L_\varepsilon w)\, u_{cc}} > 0.$$

If instead labor supply was fixed, $l_1 = l_2 = l$, we would of course get $L_\varepsilon = 0$ and $\eta^{\text{fix}} = -\frac{u_{ccc}}{u_{cc}}$. Hence, for additively separable preferences, there is a precautionary savings effect both when labor supply is flexible and when it is fixed. We also see that

$$\begin{aligned} \frac{\partial L_1}{\partial s} &= \frac{-w u_{cc}}{u_{ll} + w^2 u_{cc}} < 0, \\ \frac{\partial L_2}{\partial s} &= \frac{w u_{cc}}{u_{ll} + w^2 u_{cc}} > 0. \end{aligned}$$

That is, as risk increases, first period labor supply increases and average second period labor supply decreases. So there is a precautionary effect both on consumption goods and on leisure. The agent consumes less of both when future risk increases.

Here, flexible labor supply can affect the precautionary behavior through two mechanisms. When consumers decrease first period consumption, they also want to decrease first period leisure because of the wealth effect. Moreover, second period labor supply is uncertain and therefore there is an additional precautionary effect when $u_{lll} > 0$. These effects will work towards more saving in the first period but this may, depending on the utility function, decrease the need for prudent consumption behavior. If there are no wealth effects (i.e. if $L_\varepsilon = 0$), and if uncertainty is of this form, then labor supply flexibility can be ignored.[2]

[2] There are no wealth effects if utility is $u(c, l) = V[c + H(l)]$, where V and H are increasing and concave functions. This corresponds to Example 3 in Bodie et. al. (1992).

3.2 Wage uncertainty

Consider now an economy where agents face an uncertain second period wage rate. The first period wage is $w_1 = w$ and the second period wage is $w_2 = w + \varepsilon$. Let us first assume that the agent can choose his labor input. Given the time separable utility, the leisure choice in each period is again a function of contemporaneous consumption and wages,

$$u_l(c_t, l_t) = w_t u_c(c_t, l_t). \tag{2.11}$$

Define first period savings as $s = (1 - l_1) w_1 - c_1$. Second period consumption is then $c_2 = (1 - l_2) w_2 + s$. The indirect first period utility function is

$$\bar{v}(s) = u[(1 - L_1(s)) w - s, L_1(s)],$$

and the indirect second period utility is

$$v(s, \varepsilon) = u[(1 - L_2(s, \varepsilon)) w_2 + s, L_2(s, \varepsilon)]. \tag{2.12}$$

To get further, we need a relationship between saving and leisure. Let us again assume that the utility function is separable in consumption and leisure so that $u_{cl} = 0$. The intra temporal equilibrium condition (2.11) holds for all ε and s, so

$$L_\varepsilon \equiv \frac{\partial L_2}{\partial \varepsilon} = \frac{(1 - L_2) w_2 u_{cc} + u_c}{u_{ll} + w_2^2 u_{cc}},$$

and

$$L_s \equiv \frac{\partial L_2}{\partial s} = \frac{w_2 u_{cc}}{u_{ll} + w_2^2 u_{cc}}. \tag{2.13}$$

From the Envelope theorem we have,

$$v_s(s, \varepsilon) = u_c.$$

After differentiating again we get

$$v_{ss}(s, \varepsilon) = (1 - L_s w_2) u_{cc} = \frac{u_{ll} u_{cc}}{u_{ll} + w_2^2 u_{cc}}. \tag{2.14}$$

and

$$v_{s\varepsilon}(s, \varepsilon) = (1 - L_2 - L_\varepsilon w_2) u_{cc}.$$

A final differentiation with respect to ε yields

$$v_{s\varepsilon\varepsilon} = (1 - L_2 - L_\varepsilon w_2)^2 u_{ccc} - (2L_\varepsilon + L_{\varepsilon\varepsilon} w_2) u_{cc}$$

So

$$\eta^{\text{flex}} = \frac{-(1 - L - L_\varepsilon w)^2 u_{ccc}}{(1 - L_s w) u_{cc}} + \frac{2L_\varepsilon + L_{\varepsilon\varepsilon} w}{1 - L_s w}. \tag{2.15}$$

There are several mechanisms at work here. Labor supply changes due to a substitution effect, an income effect and a wealth effect. The substitution and income effects are due to fluctuations in the wage rate which changes the price on leisure relative to consumption. The wealth effect arises since prudent consumption behavior changes saving. Moreover, second period labor supply changes in response to wage shocks, so there will be a precautionary effect on leisure when $u_{lll} > 0$. Hence, first period leisure will tend to decrease and this tends to increase first period saving. However, for some utility functions, a decrease in leisure will tend to increase consumption of goods and hence decrease saving, so the total effect on saving, compared to the fixed labor supply scenario, is not clear.

If instead labor input is fixed at the certainty level from above, i.e. $l = L_1(0) = L_2(0,0)$, the indirect first period utility function is

$$\bar{v}(s) = u[(1 - l) w - s, l],$$

and the indirect second period utility is

$$v(s, \varepsilon) = u[(1 - l) w_2 + s, l].$$

Consequently,

$$v_s = u_c,$$

$$v_{ss} = u_{cc},$$

$$v_{s\varepsilon} = (1 - l) u_{cc},$$

and

$$v_{s\varepsilon\varepsilon} = (1 - l)^2 u_{ccc}.$$

Hence, for the fixed labor supply case we get

$$\eta^{\text{fix}} = -\frac{(1-l)^2 u_{ccc}}{u_{cc}}. \tag{2.16}$$

In practice, the parameters in the utility function are often chosen so that either the intertemporal elasticity of substitution or the degree of risk aversion get a plausible value. These measures will typically not be the same when labor supply is flexible as when it is fixed. Therefore, it is not straightforward to compare utility functions under these different assumptions. If the utility function is additively separable in consumption and leisure, however, the parameter choices will likely be indifferent to the assumptions about labor supply flexibility. The additively separable utility is thus a natural starting point for the analysis.

Assumption 1 *Utility is separable in consumption and leisure,*

$$u(c,l) = q(c) + r(l),$$

where q and r are increasing and strictly concave functions.

If we also assume that the income, substitution, and wealth effects on leisure cancel, we can show that for small risks, precautionary saving is higher when labor supply is flexible than when it is fixed. This statement is formalized in Proposition 1.

Proposition 1 *Assume that there is wage uncertainty, that the utility function fulfills Assumption 1, and that $L_\varepsilon = 0$. Then $\eta^{\text{flex}} > \eta^{\text{fix}}$.*

Proof. This follows immediately from (2.15) and (2.16) since $0 < (1 - L_s w) < 1$ by (2.13). ■

This Proposition applies to utility functions of the form $u(c,l) = \ln(c) + r(l)$, where r is a strictly concave function.

Proposition 2 *If Assumption 1 is fulfilled and $q(c) = \ln(c)$ then $L_\varepsilon = 0$.*

Proof. See the Appendix.

So far, I have compared savings decisions for agents who can choose labor supply to savings decisions for agents who are *restricted* not to vary labor input. The additively separable utility function allows us to also consider how changes in the intertemporal elasticity of leisure and labor input alters precautionary savings motives. Let us assume that the subutility for leisure belongs to the constant relative risk aversion class, and let $\bar{l}$ denote the leisure choice agents choose under certainty. Propositions 3 establishes that precautionary savings fall as leisure becomes less elastic in the utility function. Proposition 4 then shows that when leisure becomes totally inelastic, agents choose the same savings as when labor supply is fixed by regulation. The proofs of these propositions are in the appendix.

Assumption 2 *Assume that*

$$\begin{aligned} q(c) &= \ln(c) \\ r(l) &= \frac{b\left(l^{1-\mu}-1\right)}{1-\mu} \\ b &= \frac{\bar{l}^{1-\mu}}{1-\bar{l}} \end{aligned}$$

Proposition 3 *If Assumptions 1 and 2 are fulfilled, then*

$$\frac{\partial \eta}{\partial \mu} < 0.$$

Proposition 4 *If Assumptions 1 and 2 are fulfilled, then*

$$\lim_{\mu \to \infty} \eta^{\text{flex}} = \eta^{\text{fix}}.$$

Consider now two agents, one with flexible labor supply and one with exogenously fixed labor supply. Both have chosen the same saving in the first period, and both have the same utility function, $u(c,l) = \left[\left(c^{\alpha} l^{1-\alpha}\right)^{1-\gamma} - 1\right] / (1-\gamma)$. Then both agents have the same degree of risk aversion for second period wage fluctuations.[3] Therefore, if the utility function is calibrated to match a certain

[3] This is shown in the appendix.

degree of risk aversion, the calibration will be the same when labor supply is assumed to be endogenous as when labor supply is assumed to be fixed.

The unit elasticity of substitution class of utility functions is interesting since it allows us to relax the additive separability assumption and still show analytically that precautionary saving can be higher with flexible labor supply.

Proposition 5 *Assume that* $u(c,l) = \frac{\left(c^{\alpha}l^{1-\alpha}\right)^{1-\gamma}-1}{1-\gamma}$, $\gamma > 0$, *and that there is wage uncertainty. Further, assume that when labor supply is fixed, it is fixed at the level agents would choose under certainty. Then* $\eta^{\text{flex}} > \eta^{\text{fix}}$.

Proof. See the Appendix.

In Figure 1, we see that, for small γ, the degree of prudence decreases as γ rises although calculations in the appendix show that the degree of risk aversion is increasing in γ.[4] This is in accordance with Kimball and Weil's (1992) finding that both high intertemporal elasticity of substitution and high risk aversion imply much precautionary saving. In the present example, the intertemporal elasticity of substitution for total expenditure goes to infinity as γ approaches zero, but the relevant degree of risk aversion does not approach zero. The agent's allocation of resources over time becomes less important when the intertemporal elasticity of substitution is high, and since the agent has decreasing absolute risk aversion (at least when $\gamma < 1$), he willingly saves much in the first period to minimize second period risk.

3.3 Examples

In order to quantify the precautionary effects and to get results for other utility functions than those considered previously, we have to rely on numerical solutions. I consider three general classes of utility functions,

$$u^{A}(c,l) = \frac{\left(c^{\alpha}l^{1-\alpha}\right)^{1-\gamma}-1}{1-\gamma},$$

[4] It should, however, be noted that the relevance of the degree of prudence for precautionary saving is derived under the assumption that saving is small.

the constant elasticity of substitution class,

$$u^B(c,l) = \frac{\left[\alpha^{1-\nu}c^{\nu} + (1-\alpha)^{1-\nu} l^{\nu}\right]^{\frac{1-\gamma}{\nu}} - 1}{1-\gamma},$$

and the additively separable, constant relative risk aversion class,

$$u^C(c,l) = \frac{c^{1-\gamma}-1}{1-\gamma} + \frac{b\left(l^{1-\mu}-1\right)}{1-\mu},$$

where $0 < \alpha < 1$, $\gamma, \mu > 0$, and $\nu < 1$. In the CES utility function, $1/(1-\nu)$ is the elasticity of substitution between consumption and leisure. Recall that we have shown analytically that $s^{\text{flex}} > s^{\text{fix}}$ when there is wage uncertainty and the utility function belongs to class A.

Panels A, B, and C of Table I show results for different parameterizations of the respective utility functions when there are endowment shocks in the second period. The utility functions were calibrated with the same parameter values both when labor supply was flexible and when it was fixed. Table II shows the results for wage uncertainty. The variance of the endowment shocks is set to $\left(1-\bar{l}\right)^2$ times the variance of wage shocks so that the fixed labor supply experiments in Tables I and II are equivalent.

The tables show that allowing for labor flexibility can result in large changes in precautionary savings. For example, an agent with log utility ($\gamma = 1$ and utility class A) and flexible labor supply who is subject to wage uncertainty saves more than twice as much as an agent with fixed labor supply. The magnitudes of the precautionary effects in these experiments are small but so is the uncertainty agents face. The standard deviation of income is between 4 and 10 percent in these experiments.

Further, the tables indicate that labor supply flexibility lowers precautionary saving when there is endowment uncertainty while it increases saving when there is wage uncertainty. This has a natural interpretation. Because of the wealth effect, a negative endowment shock is offset by an increase in hours worked. But the substitution effect tends to decrease labor supply in response to negative wage shocks. Therefore, agents do not use variations in labor supply to self insure against wage risk to the same extent as they do against endowment risk.

As mentioned earlier, it is sometimes claimed that labor supply decisions can be neglected when utility is additively separable in consumption and leisure. We have seen that this claim is incorrect if we are looking at precautionary savings effects. However, consumption decisions are insensitive to labor supply decisions when utility is separable. For example, $c^{\text{flex}} = c^{\text{fix}}$ when $\gamma = 1$ and utility is of class A.

4 Uncertainty and labor supply

The previous analysis has indicated that more labor supply flexibility can increase the amount of precautionary savings. However, the exact ways in which agents use variations in labor supply to insure against shocks are diffuse since labor supply reacts to realized wage shocks. Let us therefore assume that second period labor supply must be chosen before uncertainty is resolved. Eaton and Rosen (1980) use the same timing, but they do not allow for precautionary saving. In contrast to Eaton and Rosen's analysis, we will see that increased wage uncertainty unambiguously lowers second period labor supply.

The second period wage is again uncertain, $w_2 = w + \varepsilon$, and we define savings as

$$s = (1 - l_1)\, w_1 - c_1.$$

Consequently, agents maximize

$$u(c_1, l_1) + \mathrm{E}_\varepsilon u(c_2, l_2),$$

subject to

$$c_2 = (1 - l_2)\, w_2 + s.$$

We want to see how s and l_2 respond to increased uncertainty.[5] To do this, rewrite first period consumption and leisure as functions of savings, and let σ denote the

[5] It is clear that both c_1 and l_1 decrease when savings increase.

uncertainty in w_2.[6] Then define

$$f(s) \equiv u_c\left[c_1(s), l_1(s)\right],$$

$$g(s, l_2; \sigma) \equiv \mathrm{E}_\varepsilon u_c(c_2, l_2),$$

and

$$h(s, l_2; \sigma) \equiv \mathrm{E}_\varepsilon\left[w_2 u_c(c_2, l_2) - u_l(c_2, l_2)\right].$$

The relevant first order conditions are then

$$f(s) = g(s, l_2; \sigma) \tag{2.17}$$

and

$$h(s, l_2; \sigma) = 0. \tag{2.18}$$

Proposition 6 states that agents choose more second period leisure and first period saving when wage uncertainty is increased.

Proposition 6 *Assume that the utility function is additively separable in consumption and leisure. Then, for small* σ*,*

$$\frac{dl_2}{d\sigma} > 0,$$

and

$$\frac{ds}{d\sigma} > 0.$$

Proof. See the Appendix.

By reducing second period labor supply, the agent avoids some risk, and for decreasing absolute risk aversion, increased saving also lowers risk. The agent can compensate the second period income loss (due to less work) by working more and consuming less in the first period.

[6] More precisely, σ is defined such that a higher σ implies more risk in the Rotschild - Stiglitz sence. So, if two stochastic variables have different σ, the variable with the highest σ can be constructed by adding a mean preserving spread to the other.

5 Concluding remarks

This paper has considered precautionary behavior of agents with flexible labor supply in a simple two-period model. The main insight of the paper is that agents do not choose the same savings behavior when labor supply is a choice variable as when their labor supply is fixed. We have seen that ignoring labor supply flexibility can have significant effects in this framework.

A potential problem with the comparisons conducted here is that risk may be measured differently under different assumptions about labor supply flexibility. If labor supply is believed to be fixed when it is not, uncertainty will be quantified as fluctuations in income rather than in wages. But income is then endogenous since it depends on work effort. However, this mismeasurement of risk will often reinforce the results found here since people work less in the second period when the wage increases (when $L_\varepsilon > 0$). In those cases, the idiosyncratic risk will be underestimated if labor supply incorrectly is assumed to be fixed.

Appendix A Proofs and calculations

A.1 Risk aversion for unit elasticity of substitution utility

Assume that $u(c,l) = \frac{\left(c^\alpha l^{1-\alpha}\right)^{1-\gamma}-1}{1-\gamma}$, $\gamma > 0$, and that there is wage uncertainty. Let us first look at the case with flexible labor supply. We then get the equilibrium condition

$$l_2 = \frac{1-\alpha}{\alpha}\frac{c_2}{w_2}, \tag{2.19}$$

so

$$c_2 = \alpha\left(w_2 + s\right).$$

The indirect second period utility function is then

$$v(s,w_2) = \frac{c_2^{1-\gamma}}{1-\gamma} w_2^{-(1-\alpha)(1-\gamma)} K, \tag{2.20}$$

where K is a constant.

We use (2.20), to find the degree of absolute risk aversion for agents with flexible labor supply,[7]

$$r^{\text{flex}} = -\frac{v_{\varepsilon\varepsilon}}{v_s} = \frac{\alpha^2\gamma^{\text{flex}} + \alpha(1-\alpha)}{w}.$$

An agent with labor supply fixed at $l = 1 - \alpha$ consumes

$$c_2 = \alpha w_2 + s,$$

and his degree of risk aversion is

$$r^{\text{fix}} = -\frac{(1-l)^2 u_{cc}}{u_c} = \frac{\alpha^2\gamma^{\text{fix}} + \alpha(1-\alpha)}{w}.$$

We see that the two agents have the same degree of risk aversion if they have the same γ.

A.2 Proof of Proposition 2

When $s = 0$ and $w_2 = w$, l and c are determined by the following two equations

$$\begin{aligned} l &= 1 - \frac{c}{w} \\ r'(l) &= \frac{w}{c} \end{aligned}$$

where the first equation is the budget constraint and the second equation is the first order condition for an optimum. It is immediately obvious that c/w must be constant when w changes. Hence $L_\varepsilon = 0$. ■

A.3 Proof of Proposition 3

$L_\varepsilon = 0$ by Proposition 2. Using this in (2.15) we get

$$\eta = \frac{-(1-l)^2 u_{ccc}}{(1-L_s w)\, u_{cc}} = \frac{K_1}{1 - L_s w} \tag{2.21}$$

[7] This is the risk attitude to what Drèze and Modigliani (1972) call "timeless" uncertainty, i.e. the risk agents face after having chosen first period saving. The expression is evaluated at $s = 0$.

where

$$K_1 \equiv \frac{(1-l)^2}{c} > 0.$$

In these equations, only L_s depends on γ. Hence

$$\frac{\partial \eta}{\partial \gamma} = \frac{wK_1}{(1-L_s w)^2}\frac{\partial L_s}{\partial \gamma}$$

and it is clear that

$$\text{sign}\left(\frac{\partial \eta}{\partial \gamma}\right) = \text{sign}\left(\frac{\partial L_s}{\partial \gamma}\right).$$

The derivative of second period leisure with respect to saving is given by (2.13),

$$L_s = \frac{wu_{cc}}{u_{ll} + w^2 u_{cc}}.$$

Consequently

$$\begin{aligned}\frac{\partial L_s}{\partial \gamma} &= -\frac{wu_{cc}}{(u_{ll} + w^2 u_{cc})^2}\frac{\partial u_{ll}}{\partial \gamma} \\ &= K_2 \frac{\partial\left(-\gamma B l^{-\gamma-1}\right)}{\partial \gamma}\end{aligned}$$

where

$$K_2 \equiv -\frac{wu_{cc}}{(u_{ll} + w^2 u_{cc})^2} > 0.$$

The coefficient B is defined as

$$B = \frac{l^\gamma}{1-l}$$

since $\bar{l} = l|_{s=0,w_2=w}$, so

$$\frac{\partial\left(-\gamma B l^{-\gamma-1}\right)}{\partial \gamma} = \frac{\partial\left[-\gamma l^{-1}(1-l)^{-1}\right]}{\partial \gamma} = -\frac{1}{l(1-l)} < 0.$$

This shows that

$$\frac{\partial \eta}{\partial \gamma} < 0.$$

■

A.4 Proof of Proposition 4

First note that η^{fix} is invariant to γ. This is seen from equation (2.16) which here yields

$$\eta^{\text{fix}} = \frac{(1-l)^2}{c}.$$

From (2.21) we get

$$\eta^{\text{flex}} = \frac{K_1}{1 - L_s(\gamma)\, w},$$

and (2.13) gives

$$L_s = \frac{w/c^2}{\gamma l^{-\gamma-1} + w^2/c^2}.$$

Since $0 < l < 1$, we see that $\lim_{\gamma\to\infty} \gamma l^{-\gamma-1} = \infty$ and consequently that $\lim_{\gamma\to\infty} L_s = 0$. This verifies the Proposition,

$$\lim_{\gamma\to\infty} \eta^{\text{flex}} = K_1 = \frac{(1-l)^2}{c} = \eta^{\text{fix}}.$$

■

A.5 Proof of Proposition 5

We evaluate η^{flex} and η^{fix} and show that $\eta^{\text{flex}} > \eta^{\text{fix}}$. Differentiate (2.20) to get

$$\eta^{\text{flex}} = -\frac{v_{s\varepsilon\varepsilon}(0,0)}{v_{ss}(0,0)} = \frac{[1-\alpha(1-\gamma)]\,[2-\alpha(1-\gamma)]}{\gamma w}.$$

When labor supply is fixed,

$$-\frac{u_{ccc}}{u_{cc}} = \frac{2-\alpha(1-\gamma)}{\mathrm{E}_\varepsilon c_2} = \frac{2-\alpha(1-\gamma)}{\alpha w}.$$

So, from (2.16)

$$\eta^{\text{fix}} = \frac{\alpha\,[2-\alpha(1-\gamma)]}{w}.$$

We know that $1-\alpha > 0$ and $\gamma > 0$ so

$$\frac{1-\alpha(1-\gamma)}{\gamma} > \alpha.$$

Hence $\eta^{\text{flex}} > \eta^{\text{fix}}$. ■

A.6 Proof of Proposition 6

Totally differentiate (2.17) and (2.18) to get

$$\frac{dl_2}{d\sigma} = -\frac{g_\sigma h_s + h_\sigma (f_s - g_s)}{(f_s - g_s) h_l + g_l h_s}, \tag{2.22}$$

and

$$\frac{ds}{d\sigma} = \frac{g_\sigma h_l - g_l h_\sigma}{(f_s - g_s) h_l + g_l h_s}. \tag{2.23}$$

We know that $f_s > 0$, $g_s < 0$, $g_l > 0$, $h_s < 0$, and $h_l > 0$. Since we have assumed that $u_{ccc} > 0$, we see that $u_c(c_2, l_2)$ is convex in w_2, and hence that $g_\sigma > 0$, where g_σ is the derivative of g with respect to its third argument. Uncertainty has unclear effects on h, however. If

$$(1 - l_2)\left[w_2 (1 - l_2) u_{ccc} + 2u_{cc} - (1 - l_2) u_{lcc}\right]$$

is positive, then $w_2 u_c - u_l$ is convex in w_2, and if the expression is negative, $w_2 u_c - u_l$ is concave.

For additively separable utility and small s and σ, the denominator in (2.22) and (2.23) is positive since then

$$\begin{aligned} f_s &= -g_s \\ g_l &= -w g_s \\ h_s &= -g_l \\ h_l &= w g_l - u_{ll}(l_2), \end{aligned}$$

that is,

$$(f_s - g_s) h_l + g_l h_s = 2g_s \left(w^2 g_s + u_{ll}\right) - w^2 g_s^2 = w^2 g_s^2 + 2g_s u_{ll} > 0.$$

It remains to show that $g_\sigma h_l - g_l h_\sigma > 0$. Expanding g and h around w, we get for small σ

$$g_\sigma = \chi_1 u_{ccc}$$

and

$$h_\sigma = \chi_1 w u_{ccc} + \chi_2 u_{cc}$$

where $\chi_1 > 0$ and $\chi_2 > 0$. Hence

$$\begin{aligned} g_\sigma h_l - g_l h_\sigma &= \chi_1 u_{ccc} \left(w g_l - u_{ll} \right) - g_l \left(\chi_1 w u_{ccc} + \chi_2 u_{cc} \right) \\ &= -\chi_1 u_{ccc} u_{ll} - g_l \chi_2 u_{cc} > 0. \end{aligned}$$

This shows that $\frac{ds}{d\sigma} > 0$. It then follows immediately that also $\frac{dl_2}{d\sigma} > 0$. ■

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Table I

Precautionary Effects with Endowment Uncertainty

Parameters			s^{flex}	s^{fix}	c_1^{flex}	c_1^{fix}	$\bar{c}_2^{\text{flex}}$	$\bar{c}_2^{\text{fix}}$	l_1	$\bar{l}_2$
A		γ								
		0.5	.006	.016	3.997	3.984	4.002	4.020	.600	.600
		1.0	.008	.020	3.997	3.980	4.003	4.021	.599	.600
		2.0	.012	.025	3.995	3.975	4.005	4.024	.599	.601
		5.0	.025	.037	3.990	3.963	4.009	4.037	.598	.601
		10.0	.046	.057	3.981	3.943	4.017	4.058	.597	.602
B	ν	γ								
	0.0	1.0	.008	.020	3.997	3.980	4.003	4.020	.599	.600
	−2.0	1.0	.006	.026	3.997	3.974	4.004	4.030	.600	.601
	0.5	1.0	.009	.017	3.996	3.982	4.003	4.015	.599	.600
	0.0	0.5	.007	.019	3.997	3.980	4.002	4.017	.600	.600
	0.0	2.0	.012	.024	3.995	3.976	4.005	4.025	.599	.601
	−2.0	2.0	.011	.034	3.995	3.966	4.005	4.036	.599	.601
C	μ	γ								
	1.0	1.0	.008	.020	3.997	3.980	4.003	4.020	.599	.600
	5.0	1.0	.012	.020	3.991	3.980	4.014	4.020	.600	.600
	2.0	2.0	.011	.030	3.995	3.970	4.005	4.031	.599	.601
	5.0	2.0	.018	.030	3.989	3.970	4.012	4.031	.599	.600
	5.0	5.0	.023	.061	3.991	3.939	4.010	4.064	.599	.601

Note: $w = 10$, $\varepsilon \sim N\left[0, \left(1-\bar{l}\right)^2\right]$, and $\alpha = 0.40$ for utility function A while α depends on ν for utility function B such that consumption is 4 and leisure is .6 under certainty. s = first period saving, superscripts refer to the model with flexible and fixed labor supply respectively.

Table II

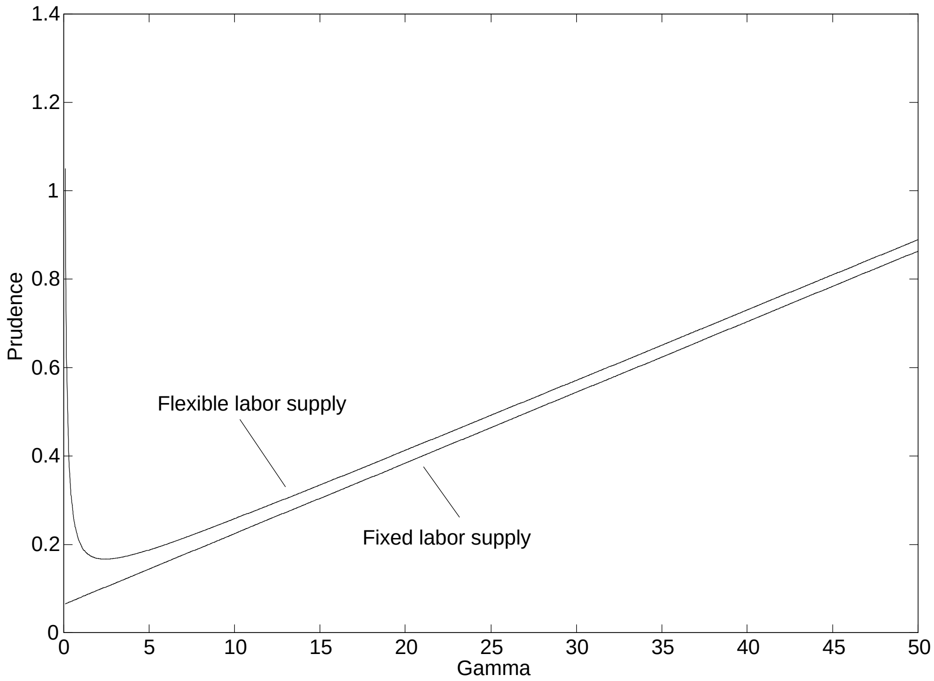
Precautionary Effects with Wage Uncertainty

Parameters			s^{flex}	s^{fix}	c_1^{flex}	c_1^{fix}	$\bar{c}_2^{\text{flex}}$	$\bar{c}_2^{\text{fix}}$	l_1	$\bar{l}_2$
A		γ								
		0.5	.068	.017	3.973	3.983	4.030	4.019	.596	.604
		1.0	.053	.021	3.979	3.979	4.020	4.020	.597	.603
		2.0	.043	.025	3.983	3.975	4.017	4.024	.597	.603
		5.0	.044	.034	3.982	3.966	4.023	4.039	.597	.603
		10.0	.068	.059	3.973	3.941	4.025	4.057	.596	.604
B	ν	γ								
	0.0	1.0	.047	.019	3.981	3.981	4.018	4.018	.597	.603
	−2.0	1.0	.048	.026	3.981	3.974	4.012	4.024	.597	.602
	0.5	1.0	.048	.016	3.981	3.984	4.030	4.014	.597	.604
	0.0	0.5	.066	.016	3.974	3.984	4.027	4.017	.596	.604
	0.0	2.0	.041	.023	3.984	3.977	4.015	4.021	.598	.602
	−2.0	2.0	.041	.032	3.983	3.968	4.010	4.032	.597	.602
C	μ	γ								
	1.0	1.0	.047	.019	3.981	3.981	4.018	4.018	.597	.603
	5.0	1.0	.024	.019	3.981	3.981	4.019	4.019	.599	.601
	2.0	2.0	.040	.028	3.984	3.973	4.011	4.028	.598	.602
	5.0	2.0	.032	.028	3.980	3.972	4.017	4.028	.599	.601
	5.0	5.0	.046	.054	3.982	3.946	4.013	4.058	.597	.602

Note: $w = 10$, $\varepsilon \sim N(0,1)$, and $\alpha = 0.40$ for utility function A while α depends on ν for utility function B, and b depends on γ and μ for utility function C, such that consumption is 4 and leisure is .6 under certainty. s = first period saving, superscripts refer to the model with flexible and fixed labor supply respectively.

Figure 1

Degree of prudence with wage uncertainty and unit elastic utility



The graph relates the degree of prudence for flexible and fixed labor supply (η^{flex} and η^{fix}) to the parameter γ in the utility function

$$u(c,l) = \frac{\left(c^{\alpha} l^{1-\alpha}\right)^{1-\gamma} - 1}{1-\gamma}$$

Chapter 3

Idiosyncratic Risk in the U.S. and Sweden: Is there a Role for Government Insurance?*

1 Introduction

Two important motivations for government taxation are that it provides insurance of individual specific income variations if private insurance markets are absent, and that it redistributes wealth from those who were born lucky to those who were not. As all feasible tax systems are to some extent distortionary, there is a trade-off between insurance and redistribution on the one hand and efficiency on the other. In some countries, such as Sweden, taxes are considerably higher than in other countries, for example the U.S.; tax receipts are approximately 60 percent of GDP in Sweden and 30 percent of GDP in the U.S. Can these differences in tax levels be motivated by differences in income distributions and income risks? Obviously, there are other reasons for government taxation than those mentioned. A more interesting question is how much government taxation is motivated by insurance and redistribution arguments.

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There are two main purposes of this paper. The first is to estimate the degree of individual specific income risk in Sweden and the U.S., and the second is to investigate to what extent government insurance via taxes and transfers should be provided. To quantify the degree of idiosyncratic risk in the respective countries, we use micro data on wages and hours worked. The estimated wage processes are then used to parameterize a general equilibrium model, in which labor supply is endogenous and agents are subject to a no-borrowing constraint. We assume that the government uses proportional labor income taxes to redistribute income among agents, and that the government wishes to maximize the ex ante utility of agents.

The wage processes are found to be highly persistent in both countries, especially in the U.S. The variance of temporary as well as permanent wage shocks is also higher in the U.S. Consequently, the wage uncertainty in the U.S. seems to dominate that in Sweden by any measure.

In the absence of tax distortions, it would be optimal for the government to redistribute almost all income equally across agents. However, we find that distortions are significant. When we calibrate the model with the estimated wage processes, the optimal size of government insurance programs amounts to labor income tax rates of 3 percent in Sweden, and 27 percent in the U.S. for our baseline calibration. The welfare loss of having no government insurance programs instead of the optimal level are 0.1 percent of annual consumption in Sweden and 5.6 percent in the U.S. The results are sensitive to the parameterization of the utility function. For the alternatives we consider, the optimal tax rate varies between 0 and 14 percent in Sweden and between 21 and 38 percent in the U.S.

The calibrated models also imply Laffer curves. These curves are of separate interest since there may be reasons for taxation in addition to the insurance motive, for example the provision of public goods. We find that the Laffer curves peak when tax rates on labor income are high, approximately 60 percent or more. As a fraction of total production, taxes levied are then around 40 percent. The shape of the Laffer curve depends on the labor-supply elasticity but seems to be invariant to a variety of other changes in parameter values and specifications of the model.

Our paper is closely related to that of Aiyagari and McGrattan (1997). They consider the welfare effects of government debt in a model where agents face idiosyncratic and uninsurable wage uncertainty and are subject to a no-borrowing constraint. Government debt increases the liquidity in the economy and effectively loosens the borrowing constraint for individuals, but it also has negative side effects. Distortive taxation is needed to finance interest payments, and debt crowds out accumulation of real capital and hence lowers production in the economy. For their benchmark calibration of the model, Aiyagari and McGrattan find the optimal level of government debt in the U.S. to be 2/3 of GDP. The income tax rate needed to sustain this debt is approximately 8 percentage points higher than in the economy with no debt. However, the welfare loss of having no debt at all instead of the optimal level would be less than 0.1 percent of consumption. Recent work by Flodén (1999) confirms that, in providing insurance, government debt is a weak instrument compared to direct transfers. In addition to allowing for government debt and not considering variations in the transfer level, the key difference from our paper is that Aiyagari and McGrattan use a considerably less persistent and slightly less volatile wage process than what we found in U.S. data.

The persistence and magnitude of wage shocks is indeed central for our results. Much previous work, for example Heaton and Lucas (1996), Aiyagari (1994), Aiyagari and McGrattan (1997), and Krusell and Smith (1997, 1998), has built on less volatile income or wage processes. In these papers, the effects of uninsurable idiosyncratic risk are in most cases small. When the persistence of shocks is low, a small buffer of wealth offers good insurance against bad outcomes, and most agents are able to build up such a buffer. Heaton and Lucas estimate the AR(1) coefficient to be .53 in annual U.S. income. Our estimation, and similar estimations by Card (1991), Hubbard, Skinner, and Zeldes (1994), and

Storesletten, Telmer, and Yaron (1997), result in higher persistence – we estimate a coefficient of .91 for the U.S. and .81 for Sweden.

There are two main differences between Heaton and Lucas' approach and that of the latter papers. First, Heaton and Lucas remove the mean from individual income series, while we estimate permanent wage differences based on observable characteristics such as education, occupation, and age. One has to take a position on what uncertainty agents face and what information agents have about their own level of productivity. Consider two agents with similar background in the beginning of the sample. One gets the income series 10, 11, 12, while the other gets the income series 10, 11, 6. Could the bad third-period outcome have happened to the first person, or is there an inherent difference between these two individuals? If there is such a difference, did the agents know about it in the first period? Second, the latter papers allow for measurement error in wage and income data. If measurement errors exist but are neglected, the estimated persistence will be downward biased.

Hansen and İmrohoroğlu (1992) explored the potential benefits of unemployment insurance in an economy where agents are subject to unemployment risk. The capital stock is exogenous, as is the working time of the employed. They find that unemployment insurance has positive welfare effects, as long as unemployed can be forced to accept all job offers. The result in that setting is not surprising since neither taxes nor unemployment benefits have any distortionary effects. Hansen and İmrohoroğlu also consider the case where unemployed can, with some probability, reject job offers but still keep their benefits. Allowing for these moral hazard considerations, the welfare gains of unemployment insurance become small or even negative.

Our paper is different from Hansen and İmrohoroğlu's in several ways. First, the uncertainty and heterogeneity we consider is richer and more important. Hansen and İmrohoroğlu only allow for two different income states, employment and unemployment, and the amount of persistence in this process is negligible.

For example, the expected earnings in a six-week period one year from now for an unemployed worker is 99.7 percent of the expected earnings of an agent who is employed today, if unemployed agents accept all job offers. Second, the possible distortions from the insurance programs in the two papers are quite different. In our paper, since the capital stock is endogenous, government policy will affect the return agents get on their private savings. As the insurance program becomes more extensive, savings fall and the interest rate increases. The cost of self insurance is then effectively reduced. Moreover, we allow for taxes and transfers having effects on labor supply, not only on the extensive margin, but also on hours worked for those who work. On the other hand, we do not allow for an explicit unemployment state, as Hansen and İmrohoroğlu do.

Some important assumptions underlying our study are worth commenting on. We abstract from aggregate uncertainty. The motivation for doing so is that a number of studies, for example İmrohoroğlu (1989), and Krusell and Smith (forthcoming), indicate that aggregate uncertainty is negligible in this setting. Also, the estimation results in Heaton and Lucas (1996) show that aggregate shocks only account for a few percent of the variability in household income.

We rule out private insurance contracts by assumption.[1] This market failure can be motivated by assuming that agents cannot observe each others' income. The government, on the other hand, is assumed to observe agents' income but not their productivity. Moreover, the government, contrary to private institutions, can force agents to participate in programs that have negative expected value for specific individuals. It should also be pointed out that the intention of this paper is not to look for efficient contracts and redistribution schemes. It is, for example, possible that it would be more efficient to condition tax rates and transfers on the income that agents have. Thus when we use the phrase "optimal tax", we do not mean this in a strict sense.

We do not explicitly allow for unemployment when estimating the wage pro-

[1] There is a significant literature studying such contracts in models with information asymmetries. Recent contributions are Atkeson and Lucas (1995) and Cole and Kocherlakota (1998).

cess. Instead, we assume that log productivity (that is, the log of the relative wage) follows an AR(1) process, but we have in mind that individuals with low productivity are unemployed. However, unemployed workers need not be completely unproductive. There are, for example, opportunities for home production or informal services. Consequently, we believe that an "unemployed" person with no accumulated wealth and no or very low guaranteed income will spend much of his time on some kind of working activity.

The structure of the paper is as follows. In the next section, we outline the model, describe how to parameterize it, and how to compute the equilibrium. The data and the strategy used to estimate the wage processes in Sweden and the U.S. are then presented in Section 3, together with the results of these estimations. In Section 4, we present results for the optimal tax level, Laffer curves, and asset distributions implied by the model. In Section 5, we try to assess how sensitive the results are to parameter choices. We also consider some changes in the specification of the model. Finally, Section 6 concludes.

2 The model

Consider an economy with a continuum of ex ante identical agents. Each year a fraction γ of the agents dies and new agents with no asset holdings enter the economy. Each agent is endowed with a level of productivity, $q_t^i = e^{\psi^i + z_t^i}$, where ψ^i is a permanent component and z_t^i a temporary component. The temporary component evolves stochastically over time according to the process

$$z_t^i = \rho z_{t-1}^i + \varepsilon_t^i, \tag{3.1}$$

where ρ determines the degree of persistence in the temporary productivity shocks. The permanent component ψ^i and the temporary shock ε^i are both assumed to be *iid* normally distributed with mean zero and variance σ_ψ^2 and σ_ε^2 respectively. Hence, the lower bound of the possible realizations of the productivity level is zero.

Each agent is also endowed with one unit of time, which is divided between labor, h, and leisure, l. There is no aggregate uncertainty in the economy. The interest rate, the wage rate, and the aggregate labor supply and capital stock will therefore be constant. The government insures agents by transferring b to each agent in every period.[2] These transfers are financed by a proportional tax on labor income. An agent's disposable resources are then

$$y_t^i = b + (1-\tau)\, w q_t^i h_t^i + (1+r)\, a_t^i,$$

where τ is the tax rate and $(1+r)\, a_t$ is the agent's asset holdings in the beginning of the period. The agent's budget constraint is

$$c_t^i \leq y_t^i - \hat{a}_{t+1}^i, \tag{3.2}$$

where $\hat{a}_{t+1}^i$ is the assets the agent chooses to hold for the next period.

In the beginning of a period, after new agents are born, a fraction γ of the population is randomly picked to be heirs to the deceased agents. The wealth of the deceased agents is then evenly distributed among the heirs.[3] Let g_t^i denote agent i's received bequests in period t, and let $\bar{a}$ denote the average wealth of an agent. Then $g_t^i = \bar{a}$ with probability γ and $g_t^i = 0$ with probability $1-\gamma$.

A crucial assumption in the model is that agents are subject to a no-borrowing constraint, i.e. that $\hat{a}_t \geq 0$. This assumption is not entirely ad hoc. If government transfers cannot be used as a security for loans, the lower bound on the present value of future incomes is zero.[4] In that case there is no positive debt which an agent can repay for sure.[5]

Let s_t^i denote the exogenous productivity state of agent i, $s_t^i = \left(\psi^i, z_t^i\right) \in \mathbf{S}$. The agents' asset holdings are restricted to be in the interval $\left[0, \bar{A}\right] = \mathbf{A}$, where $\bar{A}$ is chosen high enough to never be a binding restriction. Further, let $\lambda(a, s)$ be the measure of agents, and normalize the mass of agents to unity.

[2] A more efficient redistribution scheme would condition transfers on agents' productivity level, but we assume that q is unobservable to the government.

[3] This is similar to Huggett's (1996) "accidental bequests".

[4] Alternatively, the transfer can be in a nontradable form.

[5] See Aiyagari (1994) for a discussion of this.

Agents maximize their expected life-time utility,

$$U_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} (1-\gamma)^t \beta^t u\left(c_t^i, l_t^i\right),$$

where β is the time discount rate. The Bellman equation to the consumer's problem is then

$$v\left(a_t^i, s_t^i\right) = \max_{\left\{\hat{a}_{t+1}^i, h_t^i\right\}} u\left(c_t^i, l_t^i\right) + (1-\gamma)\beta \mathrm{E}\left[v\left(a_{t+1}^i, s_{t+1}^i\right) | \hat{a}_{t+1}^i, s_t^i\right] \tag{3.3}$$

subject to (3.2), and

$$h_t^i + l_t^i = 1,$$

$$a_t^i = \hat{a}_t^i + g_t^i,$$

$$h_t^i \geq 0,$$

$$\hat{a}_{t+1}^i \geq 0.$$

Each period the government has tax incomes given by

$$T(\tau, b) = \int_{\mathbf{A}\times\mathbf{S}} \tau w q(s) h(a,s) \, d\lambda,$$

where h is the agent's decision rule for labor supply, and $q(s)$ is the productivity level associated with state s. The government makes a lump sum transfer, b, to all agents. Its per period expenses are thus

$$G(b) = b.$$

There is a continuum of firms which have Cobb-Douglas production functions and behave competitively in product and factor markets. Let K denote the aggregate capital stock and H the aggregate labor supply in efficiency units, i.e. $H = \int q(s) h(a,s) \, d\lambda$. Aggregate production is then given by

$$F(K,H) = K^{\theta} H^{1-\theta}.$$

Finally, let δ denote the depreciation rate of capital.

2.1 Equilibrium

A stationary equilibrium of this economy is given by (i) a constant tax rate τ and level of transfers b, (ii) a constant interest rate r and wage rate w, (iii) time invariant decision rules for agents' asset holdings, $\hat{a}_{t+1}^{i} = \hat{a}'\left(a_t^i, s_t^i; \tau, b, r, w\right)$, and hours worked, $h_t^i = h\left(a_t^i, s_t^i; \tau, b, r, w\right)$, (iv) a measure of agents over the state space, $\lambda\left(a, s\right)$, (v) aggregate values for asset holdings, $A\left(\tau, b, r, w\right) = \int \hat{a}'\left(a, s\right) d\lambda$, and for the number of efficiency hours worked, $H\left(\tau, b, r, w\right) = \int q\left(s\right) h\left(a, s\right) d\lambda$, such that the following equilibrium conditions are fulfilled:

- The decision rules solve agents' maximization problem, given by (4.3).

- Tax revenues equal government expenses,

$$T\left(\tau, b, r, w\right) = G\left(\tau, b, r, w\right).$$

- Factor markets clear,

$$r = F_K\left(K, H\right) - \delta,$$

$$w = F_H\left(K, H\right).$$

- Aggregate supply of savings is equal to firms' demand for capital,

$$\left(1 + \gamma\right) A\left(\tau, b, r, w\right) = K\left(\tau, b, r, w\right).$$

- The measure of agents over the state space is invariant, i.e.

$$\lambda\left(\mathbf{a}, \mathbf{s}\right) = \int_{\mathbf{A}\times\mathbf{S}} P\left(a, s, \mathbf{a}, \mathbf{s}\right) d\lambda,$$

  for all $\mathbf{a} \times \mathbf{s} \subseteq \mathbf{A} \times \mathbf{S}$. The transition function P is the probability that an agent with state (a, s) will have a state belonging to $\mathbf{a} \times \mathbf{s}$ next period,

$$P\left(a, s, \mathbf{a}, \mathbf{s}\right) = \int_{\mathbf{s}} \{\left(1-\gamma\right)^2 \mathcal{I}\left[\hat{a}'\left(a, s\right) \in \mathbf{a}\right] + \left(1-\gamma\right)\gamma\mathcal{I}\left[\hat{a}'\left(a, s\right) + \bar{a} \in \mathbf{a}\right] + \gamma\left(1-\gamma\right)\mathcal{I}\left[0 \in \mathbf{a}\right] + \gamma^2\mathcal{I}\left[\bar{a} \in \mathbf{a}\right]\}\Gamma\left(s, ds'\right),$$

  where $\mathcal{I}$ is an indicator function and $\Gamma\left(s, \mathbf{s}'\right)$ is the probability that the exogenous state next period belongs to $\mathbf{s}' \subseteq \mathbf{S}$, given that it is s today.

2.2 Computation of equilibrium

To find the agent's decision rules for saving and labor supply, we discretisize the state space and make a piecewise linear approximation of agents' decision rules over this.[6] To solve for the equilibrium, we use an algorithm inspired by Huggett (1993) and Aiyagari (1994). The algorithm consists of the following steps: Fix the tax rate, τ, and guess an interest rate, r, and the average efficiency hours of labor supply, $\hat{H}$. Then solve for the wage per efficiency unit of labor as a function of r and $\hat{H}$, and calculate the transfer level implied by government budget balance, by setting $b = \tau \hat{H} w$. The agents' decision rules are then solved for and average asset holdings and efficiency hours worked are calculated from simulations.[7] If the implied aggregate saving of agents does not equal firms' demand for capital, or if the implied labor supply is different than the guess, then make new guesses and start over. If both equalities hold, the equilibrium of the economy with tax rate τ has been found.

2.3 Parameterization

For our baseline calibration, the agents' utility function is assumed to be in the class of CES utility functions with unit elasticity of substitution between consumption and leisure, i.e.

$$U\left(c_t, l_t\right) = \frac{\left(c_t^{\alpha} l_t^{1-\alpha}\right)^{1-\mu}}{1-\mu}.$$

[6] More precisely, we solve the Euler equation by fitting a cubic spline between gridpoints. In the simulations, the decision rules for asset holdings are approximated with piecewise linear functions. Consumption and labor decisions are solved as functions of asset choices and are therefore allowed to be nonlinear between gridpoints. The state space is approximated by a grid consisting of 50 values for asset holdings, one high and one low value for the permanent shock, and 11 values for the temporary productivity level. The AR(1) process for productivity is approximated with the algorithm by Tauchen (1986). We use a spread of $\pm 3\sigma_{\varepsilon} / \left(1-\rho^2\right)^{1/2}$ for the productivity grid. The step size in the grid for asset holdings is increasing in wealth.

[7] We simulate an economy populated by 100 agents with low permanent productivity and 100 agents with high permanent productivity for 1500 periods. When one agent dies, he is replaced by a new agent with no accumulated wealth. The initial productivity of this agent is drawn from the stationary distribution of the productivity process. We discard the first 500 periods and use the remaining 200,000 observations to calculate statistics for the economy.

This utility function has been extensively used in the real business cycle literature and it is consistent with the observation that hours worked have remained more or less constant although real wages have increased sharply the last century. However, evidence from micro studies (for example, MaCurdy, 1981, and Altonji, 1986) indicates that the intertemporal elasticity of labor supply is smaller than what is implied by this specification of utility. When doing sensitivity analysis, we consider a utility function with less elastic labor supply.

The parameter α is set to 0.50. This implies that the average time an agent spends in market activities is close to 0.50 if there is no income taxation. For positive tax rates, the agent will on average choose to work less. The time discount rate, β, is set to 0.9796, and the death probability, γ, to 2 percent. Hence, the average length of an agent's work life is 50 years and the effective time discount rate is 0.96. The inverse of the intertemporal elasticity of substitution, μ, is set to 2.

On the production side of the economy, the capital share, θ, is set to 0.36 and the depreciation rate of physical capital, δ, is set to 8 percent per year.

The parameters ρ, σ_ψ^2, and σ_ε^2 in the productivity process are estimated in the next section.

3 Data and estimation

In this section, we discuss the data sets for the U.S. and Sweden, and how we estimate the productivity processes in (3.1) on the data for the two countries. Our measure of productivity, which captures the degree of individual specific risk in the model, is an agent's hourly wage rate relative to all other agents.

3.1 Data

We use the Panel Study of Income Dynamics (PSID) data set for 1988 to 1992 to estimate ρ, σ^2_ψ, and σ^2_ε for the U.S.[8] For Sweden we use the Household Income Survey (HINK) for the years 1989, 1990 and 1992. HINK is a two-year overlapping household panel collected by Statistics Sweden, but in 1992 the collected panel is partly the same as in 1989 and 1990.

Our measure of productivity is a worker's hourly wage rate relative to all other agents. To obtain these data, we proceed as follows: For the U.S., we only look at individuals who were heads of the same household in the 1988 to 1992 surveys, and who were in the labor force (working, unemployed or temporarily laid off) all of these years. To avoid problems with oversampling of poor people in the PSID data set, we exclude people stemming from the Survey of Economic Opportunity sample. We also exclude people for whom relevant data on labor supply and earnings are of poor quality (major assignments or top-coding have been done). For Sweden, we look at adults who remained in the same household and who were in the labor force all of these years.[9]

The measure of the hourly wage which interests us is one which will hold for a wide range of hours worked for a specific individual. For example, someone who was unemployed 1000 hours in a year and worked 900 hours at the wage rate 8 dollars per hour is not assigned a wage of 8 dollars per hour but rather $8 \times 900/1900 = 3.79$ dollars per hour. Of course, unemployment is to some extent voluntary since most people could get some job at some small but positive wage rate. We will not control for this problem of inference when estimating the wage process. To avoid some of the worst problems, however, we assume that nobody has a wage rate less than ten percent of the average wage. This assumption

[8] The reason for not using a longer period is that the sample size becomes considerably smaller. The period 1988-1992 is chosen to match the Swedish data period.

[9] We include all adults for Sweden, and not only the heads of households, since there is no good definition of "heads" in the HINK database and since it is very common in Sweden that both men and women in a household participate in the regular labor market. Consequently, the share of women is higher in the Swedish sample.

also captures our belief that all agents have some productivity, although some activities are unobservable in data.

For the U.S., we calculate work hours supplied as the sum of the variables hours worked, hours in unemployment and work hours lost due to illness. These are directly observable in the PSID. For Sweden, we calculate work hours supplied as the sum of the variables hours worked and work hours lost due to illness, which are directly observable in the HINK. To this sum, we then add the estimated time in unemployment, since time spent in unemployment is not directly observable in the HINK.[10]

For people spending most of their time out of the labor force, it is difficult to infer the wage they would get if working more. Therefore, all agents with less than 1000 work hours supplied are excluded from the sample. The hourly wage rates in a year for the 1789 and 2856 persons remaining in the sample for the U.S. and Sweden respectively are then computed as the wage sums divided by the total work hours supplied.[11]

We are only interested in fluctuations in relative wages. Therefore, we remove year effects in the data by expressing agent i's hourly wage rate as a fraction of the average hourly wage rate in that year, and we denote this by w_t^i.

Descriptive statistics for the constructed relative hourly wages are reported in Table 1. For information, we also include the average hourly wage $\bar{W}$ in USD for the U.S. and in Swedish Kronor (SEK) for Sweden in the Table. We see that the variability in the relative wage series is larger in the U.S. than in Sweden, and slightly increasing over time in both countries. The minimum relative wage is 0.10 for all years as a consequence of our assumption that no individual has a wage lower than ten percent of the average. However, it should be noted that

[10] The estimated time in unemployment is an increasing function of the unemployment benefits such that the total sum of hours worked for an individual who has received unemployment benefits is set equal to the stipulated work time in Sweden, which presently is 2080 hours per year.

[11] All the definitions of variables and the data programs are provided in an appendix which is available on request from the authors. However, the HINK data set is not available upon request without a permission from Statistics Sweden.

this adjustment has been made for very few individuals.[12]

3.2 Estimation

Taking logarithms of the data, we now observe $x_t^i \equiv \ln w_t^i$ for $t = 1988$ to 1992 in the U.S. and $t = 1989, 1990$ and 1992 for Sweden. We want to estimate the process

$$\begin{aligned} x_t^i &= \psi^i + z_t^i + \xi_t^i, \\ z_t^i &= \rho z_{t-1}^i + \varepsilon_t^i. \end{aligned} \tag{3.4}$$

where we allow for a measurement error ξ and where $\psi^i + z^i$ is the logarithm of the wage rate for agent i, relative to all other agents. Both ε and ξ are assumed to be identically and independently distributed over time and across individuals.

Since our data series are short, we do not try to estimate ψ^i directly from each individual's data. Instead, we assume that the permanent wage differences can be captured by individual specific characteristics such as age, education and occupation. Hence, we estimate

$$x_{1988}^i = \varphi_1 + \varphi_2 AGE_i + \varphi_3 (AGE_i)^2 + \varphi_4 DMALE_i + \varphi_5 EDUC_i + \boldsymbol{\varphi_O} \mathbf{OCC}_i + \upsilon_{1988}^i \tag{3.5}$$

for the U.S. with OLS where AGE is the individual's age, $DMALE$ is a dummy for the individual's gender, $EDUC$ is the agent's number of years spent in school, and $\mathbf{OCC}_i = [OCC_{1,i} \ ... \ OCC_{8,i}]^T$ are occupation dummies.

For Sweden, we estimate

$$\begin{aligned} x_{1989}^i = {} & \varphi_1 + \varphi_2 AGE_i + \varphi_3 (AGE_i)^2 + \varphi_4 DMALE_i + \boldsymbol{\varphi_E} \mathbf{EDUC}_i \\ & + \boldsymbol{\varphi_O} \mathbf{OCC}_i + \upsilon_{1989}^i \end{aligned} \tag{3.6}$$

where $\mathbf{EDUC}_i = [EDUC_{1,i} \ ... \ EDUC_{3,i}]^T$ is a vector containing dummies for the agent's education level, and $\mathbf{OCC}_i = [OCC_{1,i} \ ... \ OCC_{4,i}]^T$ is a vector containing

[12] In the U.S., X^i was adjusted upwards to 0.10 for 19, 18, 20, 31 and 28 individuals in 1988, 1989, 1990, 1991 and 1992 respectively. For Sweden, X^i was set to 0.10 for 6, 10, and 26 individuals in 1989, 1990 and 1992. Changing the minimum relative wage assumption to 0.05 has no impact on the results.

occupation dummies. The variables considered in the regressions above are similar to those used by, for example, Blau and Kahn (1995) and Edin and Holmlund (1995). The estimation results for (3.5) and (3.6) are reported in Table 2.

As seen from Table 2, most of the variables are highly significant and the F-statistics are satisfactory both for the U.S. and for Sweden. The adjusted r-squares are reasonably high and similar for both countries. All the estimated parameter values are also reasonable. The point estimates for gender and age in Sweden are of the same magnitudes as the ones presented in Edin and Holmlund's (1995) wage regressions.

We use the regression results from Table 2 to calculate estimates of the permanent wage component, $\hat{\psi}^i = \hat{x}^i_{1988}$ in the U.S. and $\hat{\psi}^i = \hat{x}^i_{1989}$ in Sweden, and then to calculate the variance of these differences. For the U.S., we get $\sigma^2_\psi = 0.1175$, and for Sweden we get $\sigma^2_\psi = 0.0467$. Hence, there is more wage inequality in the U.S. than in Sweden in the sense that permanent wage differences between individuals are larger.

To extract the risk which remains for individuals in the U.S. after permanent differences have been removed, we construct the variable $\tilde{x}^i_t \equiv x^i_t - \hat{\psi}^i$ for $t = 1988, ..., 1992$. For Sweden, we construct the variable $\tilde{x}^i_t \equiv x^i_t - \hat{\psi}^i$ for $t = 1989, 1990$ and 1992. Summary statistics for the transformed relative wage variables are reported in Table 3. A comparison of the figures reported in Table 1 and Table 3 reveals that the variability in the data, quite naturally, becomes lower for both countries after the systematic factors have been removed from the data. We also see that there still is a slight increase in wage variability over time.

Finally, we use $\tilde{x}^i_t$ in (3.4) to construct the following unconditional moment conditions

$$\begin{aligned} \mathrm{E}\left[\left(\tilde{x}^i_t\right)^2\right] - \frac{\sigma^2_\varepsilon}{1-\rho^2} - \sigma^2_\xi &= 0, \qquad (3.7) \\ \mathrm{E}\left[\tilde{x}^i_t \tilde{x}^i_{t-s}\right] - \rho^s \frac{\sigma^2_\varepsilon}{1-\rho^2} &= 0 \end{aligned}$$

in order to estimate ρ, σ^2_ε, and σ^2_ξ for the U.S. and Sweden with the general method of moments. Since we have observations from five periods in the U.S., (3.7) implies that we can use 15 moments. For Sweden, (3.7) implies that we can use 6 moments. Since we have more moments than estimated parameters, the model is overidentified, and we use Hansen's (1982) χ^2-test to test the overidentifying restrictions. However, it is well known that Hansen's test may fail (see Newey, 1985). Therefore, the p-values for Hansen's test, reported in Table 4, were generated with a Monte Carlo simulation.[13]

The GMM estimation results are reported in Table 4. We see that the relative hourly wage series are highly persistent, especially in the U.S. Moreover, the variance of temporary shocks is considerably higher in the U.S. than in Sweden. Consequently, the wage risk that agents face after having observed their permanent productivity level is higher in the U.S. The estimates of ρ and σ_ε^2 are precise for both countries. As indicated by the simulated p-values, one possible shortcoming is that the overidentifying restrictions do not seem to hold, in particular not for Sweden. One reason for this result might be that the estimated AR(1)-process for the agent's productivity process is a too crude approximation of reality.[14]

To sum up, we have found that individuals in the U.S. are subject to more wage inequality as well as more wage uncertainty. The estimated variance of permanent log wage differences is 0.1175 in the U.S. and 0.0467 in Sweden. The estimated variance of temporary log wage shocks is 0.0426 in the U.S. and 0.0326 in Sweden, and temporary shocks are more persistent in the U.S., with the estimate of ρ equal to 0.9136 against 0.8139 in Sweden.

The findings for the U.S. wage process resemble those in Card (1991). He

[13] In the Monte Carlo study, we have simulated the process $x_t^i = z_t^i + \xi_t^i$ where $z_t^i = \rho z_t^i + \varepsilon_t^i$, using $\hat{\rho}$, $\hat{\sigma}_\varepsilon^2$ and $\hat{\sigma}_\xi^2$ from Table 4, and calculated χ^2 from these simulated series.

[14] However, if we assume that all unemployment is voluntary (which here in practice means that we do not add time in unemployment to hours worked in the calculation of hourly wages), the estimated σ_ψ^2, ρ and σ_ε^2 are practically unchanged ([0.1075, 0.9165, 0.0379] and [0.0421, 0.8545, 0.0227] for the U.S. and Sweden respectively). But the χ^2-statistics are now changed to 15.02 and 12.12 with p-values 0.27 and 0.03 respectively. Thus, we can no longer clearly reject the model. We therefore conclude that the estimates for the parameters σ_ψ^2, ρ and σ_ε^2 seem to be robust, but that Hansen's χ^2-statistic seems to be sensitive to the data generation.

estimated a similar wage process for the U.S. based on men in the PSID from 1969 to 1979. The estimated persistence was 0.886 while the estimates of variances were 0.124, 0.027, and 0.039, for permanent shocks, temporary shocks, and measurement errors, respectively.

4 Optimal tax level, Laffer curves and asset distributions

4.1 Optimal tax level and Laffer curves

To find the optimal tax level, we solve the model for tax rates between 0 and 65 percent, with increments of 1 percentage point, and look for the tax rate that maximizes the average utility of agents in the economy, $\bar{u}$. Equilibrium outcomes for some selected tax rates are shown in Tables 5a and 5b. As a reference, we also report the outcome we would get if agents were provided with full insurance at zero tax rates.[15]

For the baseline calibration, we find the optimal tax rate to be 27 percent for the U.S. and 3 percent for Sweden. This result is visualized in Figure 1, where the average utility is increasing up to $\tau = 0.27$ in the U.S. but decreasing in Sweden for all τ larger than 0.03. The relatively large differences between the U.S. and Sweden are not surprising, given the estimated wage processes. Some experiments show that both the differences in variances of wage shocks and the difference in persistence of these shocks are quantitatively important. The optimal tax rate in the U.S. falls to 16 percent if ρ is set to the value estimated from Swedish data, and it falls to 23 and 21 percent, respectively, if σ_{ε}^2 and σ_{ψ}^2 are set to the Swedish counterparts (while leaving the other parameters unchanged).

In the U.S., the welfare gain of having the optimal level of government insurance instead of no insurance at all is large, while the same welfare gain for

[15] By full insurance, we mean that all agents insure before observing their first productivity level. The insurance then yields the same marginal utility of total expenditure in each state.

Sweden is negligible. We measure these gains as the percentage increase in consumption needed for agents in the no-insurance world to get the same average utility as agents in the optimal-insurance world. When doing these calculations, we fix decision rules and prices in the no-insurance world.[16] The welfare gain is 5.6 percent of yearly consumption in the U.S. and 0.1 percent in Sweden.

As seen in Tables 5a and 5b and Figure 2, the Laffer curve has its maximum at very high values of τ. The Laffer curve peaks at a tax rate of 60 percent in the U.S. and 59 percent in Sweden. Although the optimal tax rates differ significantly between Sweden and the U.S., the Laffer curves are similar. The distortive effects of income taxes do not seem to be sensitive to the amount of risk that agents face.

A couple of other features in Tables 5a and 5b are also worth noting. In an economy with a higher degree of idiosyncratic risk, aggregate output reacts more to changes in the tax rate. We see this in Figure 3, where the difference in aggregate output between the U.S. and Sweden becomes smaller as τ increases. The intuition behind this result is that an increase in the transfer level has larger insurance effects in a country with much idiosyncratic risk than in a country with little risk. As b increases, agents in the U.S. therefore reduce their holding of precautionary wealth more than agents in Sweden do.

We also note that a high degree of idiosyncratic risk is "good" for the agents if they can insure themselves against periods with low productivity and "bad" if they cannot. This result can be seen by comparing the full insurance rows and the first rows in Tables 5a and 5b. The explanations behind this result are twofold. When agents are fully insured, they are able to smooth consumption by borrowing and lending. The agents can then choose to work more when their productivity is high and less when productivity is low, and the higher the degree of idiosyncratic risk, the more agents can increase their utility by working

[16] By allowing agents to reoptimize or by increasing transfers instead of consumption, the measured welfare effects would be slightly lower.

when their productivity is high and staying at home when it is low.[17] But when asset markets are incomplete, agents can no longer smooth consumption and leisure independently. If they have little wealth and low productivity, agents must work to be able to consume. Because of the concavity of the utility function, productivity fluctuations will decrease agents' utility. Therefore, the average utility, $\bar{u}$, is higher in the U.S. than in Sweden under full insurance, but lower when asset markets are incomplete and no government insurance is provided.

When looking for the optimal tax rate, we have taken a utilitarian approach and put equal weight on every agent's utility. To understand which individuals, when considering the stationary distribution of agents, find that government transfers really matter, we have computed optimal tax rates for different percentile agents in this distribution. The main value of the experiment is that it gives a picture of inequality and a sense of which agents experience that social security really matters. The results show that government transfers, at the level suggested by the previous analysis, benefit the lowest 30 percentiles in the utility distribution. The median utility in both countries is maximized when tax rates are close to zero.

4.2 Asset distributions

To investigate the empirical validity of the calibrated model, we present distributional implications for the U.S. and Sweden in Tables 6a and 6b, respectively. It is a well known fact that models with plausible parameterizations of income processes and risk aversion have problems in generating asset and income distributions which are as skewed as in the U.S. data. This is documented in e.g. Quadrini and Ríos-Rull (1997).[18] The wealth distributions implied by our model are skewed, but not as skewed as the actual Swedish and U.S. distributions. In

[17] This mechanism is most clearly seen from the FI rows in Tables 5a and 5b, where H is significantly higher than $\bar{h}$.

[18] Examples of such studies are Aiyagari (1994) and Huggett (1996). A recent exception is the paper by Castañdena, Díaz-Giménez, and Ríos-Rull (1998). They calibrate the underlying productivity process so that asset and income distributions are matched.

particular, the model cannot generate wealth holdings that are as extreme as for the top few percent of households in the data. However, for the question we are interested in, we argue that it is most important to capture the asset and income distributions of the poor agents, because it is for these that social security really matters. The model does fairly well in this respect.

For the U.S., we report distributions for two tax rates, $\tau = 0.15$ and $\tau = 0.30$. The lower tax level implies that transfers are approximately at the U.S. postwar average of 8 percent of GDP reported in Aiyagari and McGrattan, while the higher tax rate is closer to the U.S. labor income tax rate.[19] For Sweden, we use the tax rates $\tau = 0.30$ and $\tau = 0.50$.

The tables show that asset holdings are unequally distributed, with Gini coefficients around 0.60, but still not as skewed as in the actual economies. In particular, the wealthiest agents (households) in the model are not at all as wealthy as in Sweden and the U.S. The richest one percent of agents hold 8 percent of aggregate wealth in the model, but in the U.S. they hold 29 percent of all wealth. For Sweden, the richest one percent hold 5 percent of all wealth in the model and 13 percent of all wealth in the data.

The asset distribution for the poorest agents is better matched by the model. The bottom 40 percent of agents (households) in the wealth distribution hold approximately 1 percent of the U.S. wealth in the data and 2 percent in the model. In Swedish data they hold -6 percent of all wealth and 5 percent in the model. According to Domeij and Klein (1998) there are two main reasons for the frequent measures of negative wealth holdings for Swedish households. First, the value of privately owned apartments is approximated by the taxable value, which is considerably lower than the market value. Second, students' loans are measured at the full value but human capital is not included in wealth. Considering these data problems, we think the model gives a satisfactory fit of the poor agents in the asset distribution.

[19] In the benchmark model, all tax income is used for transfers, but of course this is not the case in reality.

The earnings and income distributions for Sweden are well captured by the model, both for those in the bottom and those in the top of the distributions. The model generates too compressed distributions for the U.S., however. For example, the bottom 40 percent in the earnings distribution have only 3 percent of earnings in the data but around 10 percent in the model. In the U.S. data, entrepreneurs who report losses significantly contribute to the low earnings for the bottom percentiles in the distribution. In the model, wage rates are observable in the beginning of a period, and we do not allow for negative wages.

Maybe surprisingly, changes in tax rates have negligible effects on wealth distributions. For both countries, an increase in taxes actually increases Gini coefficients. When transfers increase, there is less need for poor agents to save for bad times, and in bad times they do not need to work as hard as when there are no transfers.

5 Sensitivity to parameter choice and model specification

In this section, we examine how sensitive the results are with respect to the most important parameters and some specific model assumptions. The results are summarized in Table 7. Since the estimated confidence intervals for ρ and σ_ε^2 were small, we do not make any sensitivity analysis for these parameters, but some of the effects of changes in ρ and σ_ε^2 were shown in Section 4.1.

5.1 The utility function

To get plausible values for hours worked we chose to set α, the weight on consumption relative to leisure, to 0.50 in the baseline calibration. We examined the effects of setting α to the more common value 0.33, although this value yields a counterfactually low labor supply in the current model. A lower α also implies a higher labor-supply elasticity. Taxes therefore become more distortive when α

is decreased. The results in Table 7 show that this effect is quantitatively small. With the lower α, the optimal tax rates fall to 23 and 1 percent in the U.S. and Sweden respectively. The peaks of the Laffer curves shift slightly to the left.

Plausible values for the intertemporal elasticity of substitution are often claimed to be in the interval $[0.2, 1]$. We considered the extreme values, $\mu = 5$, and $\mu = 1$. Not surprisingly, the chosen value for μ is important for the results obtained. If μ is increased from 1 to 5, the optimal tax rate increases from 21 to 36 percent in the U.S., and from 0 to 14 percent in Sweden.

Estimates of the wage elasticity of labor supply vary widely between studies. However, most estimates of the elasticity are less than .5 for men, and the estimated elasticity for women is typically higher than that for men – see for example MaCurdy (1981) and Altonji (1986) for estimates on U.S. data, and Flood and MaCurdy (1992) and Aronsson and Palme (1998) for Swedish estimates. As mentioned earlier, the labor-supply elasticity implied by the Cobb-Douglas utility function is higher than what was found in these studies. To allow for a less elastic labor supply, we consider the utility function

$$u(c,l) = \frac{c^{1-\mu} - 1}{1-\mu} + \Lambda \frac{l^{1-\lambda} - 1}{1-\lambda},$$

where $1/\lambda$ is the labor-supply elasticity. When this utility function is parameterized with $(\mu, \lambda) = (2, 2.5)$, τ^* increases to 38 percent in the U.S. and 10 percent in Sweden.

Although labor supply seems inelastic, microdata display considerable variability in hours worked. The evidence reported in Altonji and Paxson (1985), Abowd and Card (1989), and Card (1991) suggest that the coefficient of variation for hours worked, conditional on hours being positive, is between .25 and .40 in the U.S. Aronsson and Palme (1998) report coefficients of variation of .14 and .41 for married Swedish men and women respectively. Both utility functions considered here are consistent with these facts. For the baseline specification of the utility function, the standard deviation of changes in log hours worked is .32 in the U.S. when there is no government sector, and .46 when tax rates are 30 percent. With the new utility function these figures drop to .19 and .23 respectively. For the Swedish setup of the model, the values are .27 and .40 for the baseline calibration and .13 and .14 with the alternative utility function.

5.2 Infinitely lived agents

In the baseline calibration of the model, agents live 50 years on average, bequests are random over the life cycle, and newly born agents have no wealth. We think that this is a good way to describe reality in a parsimonious way, but the assumptions are non-standard. One might suspect that our results hinge on the poor situation for newly born agents who have not had time to accumulate a buffer of wealth. However, if we assume that agents have infinite lives ($\gamma = 0$, but the effective discount rate unchanged, $\beta = 0.96$), the optimal tax rate only falls slightly, to 23 percent and 2 percent in the U.S. and Sweden respectively.

We are a bit surprised by this small effect of changes in γ. With $\gamma = 0$, agents live for ever and hence have time to accumulate some wealth to self insure against bad times. There are then few agents who have both very little wealth and low productivity, the state which agents want to avoid almost at any cost. However, the accumulation of individual buffer stocks is inefficient in itself, and although government redistribution schemes distort labor supply, they seem to provide better insurance than private savings.

5.3 Only temporary risk

The U.S. wage process displays more temporary risk as well as more permanent inequality than the Swedish process. Which of these differences is most important for our results? Although we prefer to think of both the permanent wage differences and the temporary fluctuations as risks for which the government can provide insurance, in daily life transfers because of the former would usually be thought of as redistribution.

By ignoring the permanent wage differences in the calibration of the wage process, we get an impression of which source of risk is driving our results. We find that with only temporary wage uncertainty, the optimal tax rate is 18 percent in the U.S. while no redistribution is motivated in Sweden.

5.4 Government spending

There are other reasons for the government to levy taxes than the insurance and redistribution motive. If the government has a fixed amount of spending on public goods to finance, the distortive effects of increased taxes will be more severe since the tax base for public goods is eroded. We have tried to quantify such effects in the following way. Assume that spending on public goods is not valued by the agents, or equivalently that the utility is additively separable in private and public consumption. Fix government spending at the level it would be if all taxes levied with 10 percent income taxes were used for public consumption. Any amount of tax income the government raises above that amount is transferred in lump sums to the agents as previously.

With this assumption, we obtain lower optimal redistribution levels than in the baseline solution. For Sweden, no redistribution is motivated, and for the U.S. the optimal tax rate is 28 percent including the 10 percent base tax. Note that more than 10 of the 28 percentage points of the tax rate are used to finance government expenditures. When including the base tax in tax income, the Laffer curves shift slightly to the right. This is partly because of a wealth effect, making all agents work more when resources are wasted on government consumption, and partly because of an insurance effect - poor agents get less transfers at a given tax rate and must work harder to get a sufficient income.

5.5 Open economy

Sweden is often thought of as a small, open economy which faces a given world interest rate, but until now we have assumed that both Sweden and the U.S. are closed economies. In Table 5b, we saw that the equilibrium capital stock in

Sweden is decreasing in the tax rate. Does this mean that distortions are less important when the world capital stock is given? We conducted some experiments to answer this question. We solved the model for Sweden with the interest rate fixed at 3.12 percent, which is the equilibrium interest rate for the U.S. at a 30 percent labor income tax.[20]

The results for this scenario are similar to what we found with the original specification. The optimal tax rate increases to 6 percent and the Laffer curve peaks at tax rates close to 60 percent. The reason for the small change in the optimal insurance level is that the interest rate is not the sole determinant of the capital stock. More important is the supply of efficiency units of labor, and this supply is very sensitive to tax rates. So, although the world interest rate is given and capital is totally mobile, the equilibrium capital input in Swedish production is sensitive to changes in the tax rate.

6 Concluding remarks

We want to stress the main findings of the paper. Wage inequality and wage fluctuations seem to be important features of the economies studied, but more severe in the U.S. than in Sweden, and it seems as if agents, at least in the U.S., are willing to give up a significant amount of consumption in order to insure against this uncertainty.

One possible explanation for the results is that agents in the U.S. are less risk averse than agents in Sweden, and choose higher average wages at the price of higher wage fluctuations. This interpretation is consistent with the fact that GDP per capita is higher in the U.S. than in Sweden.

For all the specifications we have considered, the Laffer curve has peaked when tax rates on labor income have been 58 percent or higher. In our experiments,

[20] This approach could have been invalid if the Swedish interest rates in autarky had been lower than the U.S. interest rate. People in Sweden might then want to hold much wealth when the high world interest rate prevails. Consequently, even if the Swedish population is small, it could have a significant impact on capital formation.

only changes in the labor-supply elasticity matter for the shape of the Laffer curve. To claim that the Laffer curve peaks at lower tax rates, one has to believe that the elasticity of labor supply is considerably higher than what is typically estimated from data.

There are also some caveats we want to point out to the reader. First, although we look at wages before taxes and transfers, the relatively low degree of wage risk in Sweden may be a result of the big government sector. For example, a large fraction of the population work in the government sector and wage setting there seems to imply a significant amount of risk sharing. Also, many old persons who become unemployed go into early retirement and hence fall out of the labor force and our sample. Moreover, we take labor market and wage setting institutions as given. That is, we do not try to understand or explain why wage processes are different in different countries. Arguably, some of these differences are a result of government policy. If, for example, wages are a result of bargaining between unions and firms, the bargaining position of low income groups may improve relative to that of high income groups if transfers are increased. We abstract from such issues.

Second, a lot has happened in Sweden after the period examined. Unemployment has increased drastically and in particular employment in the government sector has fallen. It is therefore possible that the income risk in Sweden has increased. Third, we believe that our modeling of idiosyncratic risk is more appropriate for the U.S. than for Sweden. In Europe, the risk that agents face seems to be mainly unemployment risk, not wage risk.

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Table 1: Descriptive statistics for relative wages

	U.S.					Sweden		
Statistic	1988	1989	1990	1991	1992	1989	1990	1992
$\bar{W}$	12.48	13.31	14.11	14.79	15.60	71.20	81.42	88.83
Std(w^i)	0.64	0.62	0.65	0.66	0.71	0.40	0.40	0.45
Max(w^i)	8.18	5.53	8.27	10.11	12.14	4.63	4.53	4.82
Min(w^i)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10

Note: $\bar{W}$ is the average hourly wage in USD and SEK respectively. w^i is the relative wage, Std(w^i) the standard deviation in w^i and Max(w^i) and Min(w^i) the maximum and minimum relative wage in the constructed relative wage series in a given year.

Table 2: OLS estimation results for the initial relative wage level

U.S. - 1988			Sweden - 1989		
Variable	Estimate	p-value	Variable	Estimate	p-value
$CONSTANT$	-3.330	0.000	$CONSTANT$	-1.079	0.000
AGE	0.076	0.000	AGE	0.033	0.000
$AGE^2/100$	-0.077	0.000	$AGE^2/100$	-0.035	0.000
$DMALE$	0.272	0.000	$DMALE$	0.194	0.000
$EDUC$	0.074	0.000	$EDUC_1$	0.099	0.000
OCC_1	0.421	0.000	$EDUC_2$	0.218	0.000
OCC_2	0.320	0.000	$EDUC_3$	0.475	0.000
OCC_3	0.277	0.001	OCC_1	0.061	0.000
OCC_4	0.231	0.042	OCC_2	0.068	0.013
OCC_5	0.257	0.000	OCC_3	0.055	0.006
OCC_6	0.171	0.017	OCC_4	0.083	0.002
OCC_7	-0.558	0.000			
OCC_8	0.076	0.233			
F	59.166	0.000	F	120.238	0.000
$\bar{R}^2$	0.281		$\bar{R}^2$	0.295	
N	1789		N	2856	

Note: Dependent variables are the ratio between the hourly wage and average hourly wage in the U.S. 1988 and Sweden 1989 in natural logarithms. For the U.S., $EDUC$ is the number of years spent in school, OCC_1, ..., OCC_8 are dummy variables equal to 1 if the individual is a professional or technical worker, manager, sales worker, clerical worker, craftsman, operative, farm worker, or service worker, respectively and 0 otherwise. A dummy for unclassified occupations is excluded in the regression. For Sweden, $EDUC_1$, ..., $EDUC_3$ are dummy variables equal to 1 if the individual has between $2-3$, $3-6$ and over 6 years education after primary school respectively and 0 otherwise. A dummy for those with less than 2 years education after primary school is excluded. OCC_1, ..., OCC_4 are occupation dummies equal to 1 if the individual works in the private industry, building industry, sales sector and the communication and transport sector. A dummy variable for those who work in the public sector and in banks is excluded. Finally, $DMALE$ is a dummy variable equal to 1 if the individual's gender is male and 0 otherwise.

Table 3: Descriptive statistics for transformed relative wages

	U.S.					Sweden		
Statistic	1988	1989	1990	1991	1992	1989	1990	1992
Std$(\tilde{w}^i)$	0.57	0.56	0.59	0.59	0.64	0.31	0.30	0.37
Max$(\tilde{w}^i)$	5.73	4.37	5.79	7.09	8.51	2.95	3.27	4.10
Min$(\tilde{w}^i)$	0.09	0.08	0.08	0.08	0.08	0.10	0.07	0.07

Note: $\tilde{w}^i \equiv \exp\left(\tilde{x}^i\right)$, that is, the relative wage where the estimated systematic component due to permanent differences between individuals in the sample has been removed. Std$\left(\tilde{w}^i\right)$ the standard deviation in $\tilde{w}^i$ and Max$\left(\tilde{w}^i\right)$ and Min$\left(\tilde{w}^i\right)$ the maximum and minimum relative wage in the constructed relative wage series in a given year.

Table 4: GMM estimation results for the productivity process

	U.S.		Sweden	
Parameter	Estimate	Standard error	Estimate	Standard error
ρ	0.9136	0.0090	0.8139	0.0268
σ_ε^2	0.0426	0.0048	0.0326	0.0059
σ_ξ^2	0.0421	0.0039	0.0251	0.0046
χ_{obs}^2	23.45		46.35	
p-value	0.051		0.000	

Note: White's heteroskedasticity consistent standard errors. The p-values are simulated probabilities of obtaining a χ^2 higher than χ_{obs}^2 when the model is correctly specified.

Table 5a: Results for different tax rates – U.S.

τ	$\bar{u}$	r	K	H	Y	C	$\bar{h}$	T	T/Y
.00	-1.820	2.42	3.28	0.472	0.949	0.685	0.427	0.000	.000
.05	-1.804	2.55	3.12	0.458	0.915	0.664	0.409	0.029	.032
.10	-1.790	2.67	2.97	0.444	0.881	0.642	0.389	0.056	.064
.15	-1.781	2.79	2.82	0.430	0.847	0.620	0.369	0.081	.096
.20	-1.774	2.90	2.68	0.415	0.813	0.598	0.349	0.104	.128
.25	-1.771	3.02	2.54	0.399	0.777	0.573	0.328	0.124	.160
.30	-1.772	3.12	2.40	0.382	0.741	0.548	0.306	0.142	.192
.35	-1.777	3.23	2.26	0.366	0.701	0.523	0.285	0.158	.225
.40	-1.787	3.34	2.12	0.348	0.667	0.497	0.263	0.171	.256
.45	-1.801	3.44	1.98	0.330	0.630	0.470	0.241	0.181	.287
.50	-1.822	3.54	1.84	0.311	0.589	0.441	0.219	0.189	.321
.55	-1.849	3.65	1.70	0.291	0.549	0.412	0.197	0.193	.352
.60	-1.886	3.75	1.55	0.270	0.507	0.382	0.176	0.195	.385
.65	-1.934	3.85	1.41	0.248	0.463	0.350	0.154	0.193	.417
FI	-1.598	4.17	2.46	0.451	0.831	0.634	0.303	0.000	.000

Note: $\bar{u}$ = average utility, r = real interest rate, K = aggregate capital stock, H = aggregate efficiency units of hours worked, Y = aggregate output, C = aggregate consumption, $\bar{h}$ = average hours worked, T = total tax revenues, and FI = outcome under full insurance.

Table 5b: Results for different tax rates – Sweden

τ	$\bar{u}$	r	K	H	Y	C	$\bar{h}$	T	T/Y
.00	-1.736	3.63	2.69	0.460	0.868	0.652	0.436	0.000	.000
.05	-1.736	3.67	2.62	0.447	0.842	0.633	0.421	0.027	.032
.10	-1.737	3.73	2.50	0.433	0.814	0.613	0.406	0.052	.064
.15	-1.740	3.77	2.40	0.419	0.786	0.592	0.390	0.076	.097
.20	-1.746	3.82	2.30	0.405	0.751	0.571	0.373	0.097	.129
.25	-1.754	3.87	2.20	0.389	0.726	0.548	0.356	0.116	.160
.30	-1.765	3.91	2.10	0.373	0.695	0.525	0.338	0.133	.191
.35	-1.779	3.95	1.99	0.356	0.662	0.501	0.319	0.148	.224
.40	-1.798	3.99	1.88	0.338	0.627	0.475	0.299	0.161	.257
.45	-1.821	4.03	1.77	0.319	0.591	0.448	0.278	0.170	.288
.50	-1.851	4.07	1.65	0.300	0.554	0.420	0.257	0.177	.319
.55	-1.889	4.12	1.53	0.279	0.514	0.391	0.235	0.181	.352
.60	-1.934	4.16	1.40	0.256	0.471	0.360	0.211	0.181	.384
.65	-1.997	4.16	1.28	0.235	0.432	0.326	0.189	0.177	.410
FI	-1.651	4.17	2.47	0.454	0.836	0.638	0.391	0.000	.000

Note: See Table 5a.

Table 6a: Distributional implications – U.S.

	Gini	Percent of total			
		Bottom 40%	Top 20%	Top 10%	Top 1%
Wealth					
Actual U.S. Data	.78	1.4	79.5	66.1	29.5
Model, $\tau = 0.15$	.63	2.6	63.6	42.6	8.0
Model, $\tau = 0.30$	.64	2.2	64.4	43.2	8.0
Earnings					
Actual U.S. Data	.63	2.8	61.4	43.5	14.8
Model, $\tau = 0.15$	.48	10.9	51.3	33.2	6.1
Model, $\tau = 0.30$	.53	8.0	54.9	35.9	6.7
Total income					
Actual U.S. Data	.57	8.8	59.9	45.2	18.6
Model, $\tau = 0.15$	.39	16.3	45.8	29.3	5.3
Model, $\tau = 0.30$	.39	17.0	45.4	29.0	5.2

Note*:* U.S. data adapted from Díaz-Giménez, Quadrini, and Ríos-Rull (1997). Earnings is defined as net labor income before taxes. Total income is defined as net factor income plus transfers but before taxes. Note that U.S. data refer to households while the income process in the model is calibrated to match individual wage processes.

Table 6b: Distributional implications – Sweden

	Gini	Percent of total			
		Bottom 40%	Top 20%	Top 10%	Top 1%
Wealth					
Actual Swedish Data	.79	-6	72	49	13
Model, $\tau = 0.30$	.56	5	56	35	5
Model, $\tau = 0.50$	.57	5	56	35	5
Earnings					
Actual Swedish Data	.48	8	47	29	5
Model, $\tau = 0.30$	.38	16	42	25	4
Model, $\tau = 0.50$	.46	11	47	29	5
Total income					
Actual Swedish Data	.33	19	37	14	5
Model, $\tau = 0.30$	.26	23	36	21	3
Model, $\tau = 0.50$	.27	23	37	22	3

Note*:* Swedish data adapted from Domeij and Klein (1998). Earnings is defined as net labor income before taxes. Total income is defined as net factor income plus transfers but before taxes. Note that Swedish data refer to households while the income process in the model is calibrated to match individual wage processes.

Table 7a: Sensitivity analysis – U.S.

Parameter values								
μ	α	γ	τ^*	$\left(\frac{T}{Y}\right)^*$	r	$\frac{\bar{c}(\tau^*)}{\bar{c}(0)}$	$\tau^{\max}$	$\frac{T^{\max}}{Y}$
2.0	0.50	0.02	.27	.17	3.05	0.82	.60	.38
1.0	0.50	0.02	.21	.13	3.07	0.87	.60	.38
5.0	0.50	0.02	.36	.23	2.78	0.73	.59	.38
2.0	0.33	0.02	.23	.15	2.76	0.82	.58	.37
2.0	0.50	0.00	.23	.15	2.60	0.85	.60	.38
Utility function			.38	.24	3.37	0.89	n/a	n/a
Only temporary risk			.18	.12	2.87	0.87	.58	.37
10% base tax[a]			.18	.18	3.02	0.87	.62	.40

Note: τ^* = optimal tax rate, $\frac{\bar{c}(\tau^*)}{\bar{c}(0)}$ = average consumption when taxes are τ^* as a fraction of average consumption when there is no taxation, $\tau^{\max}$ is the tax rate which maximizes government tax income. The row 'utility function' refers to the case where utility is $\left(c^{1-\mu}-1\right)/(1-\mu)+\Lambda\left(l^{1-\lambda}-1\right)/(1-\lambda)$, $\mu=2$, and $\lambda=2.5$. [a] The base tax is included in $\tau^{\max}$ but not in τ^*.

Table 7b: Sensitivity analysis – Sweden

Parameter values								
μ	α	γ	τ^*	$\left(\frac{T}{Y}\right)^*$	r	$\frac{\bar{c}(\tau^*)}{\bar{c}(0)}$	$\tau^{\max}$	$\frac{T^{\max}}{Y}$
2.0	0.50	0.02	.03	.02	3.66	0.98	.59	.38
1.0	0.50	0.02	.00	.00	3.64	1.00	.59	.38
5.0	0.50	0.02	.14	.09	3.54	0.91	.59	.38
2.0	0.33	0.02	.01	.01	3.47	0.99	.56	.36
2.0	0.50	0.00	.02	.01	3.39	0.99	.58	.37
Utility function			.10	.06	3.76	0.98	n/a	n/a
Only temporary risk			.00	.00	3.65	1.00	.58	.37
10% base tax[a]			.00	.00	3.71	1.00	.64	.41
Open economy			.06	.04	3.12	0.97	.58	.37

Note: See Table 7a.

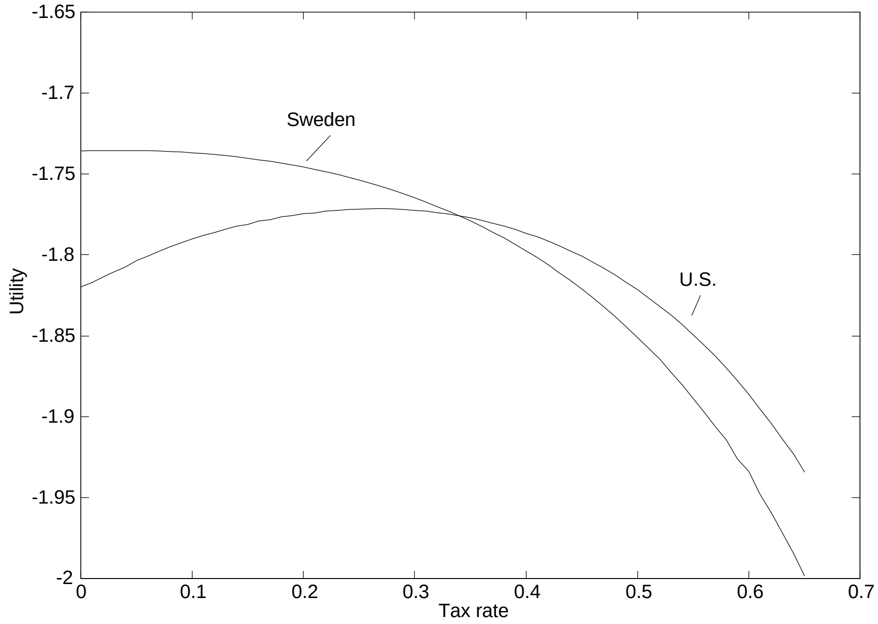

Figure 1: Average utility

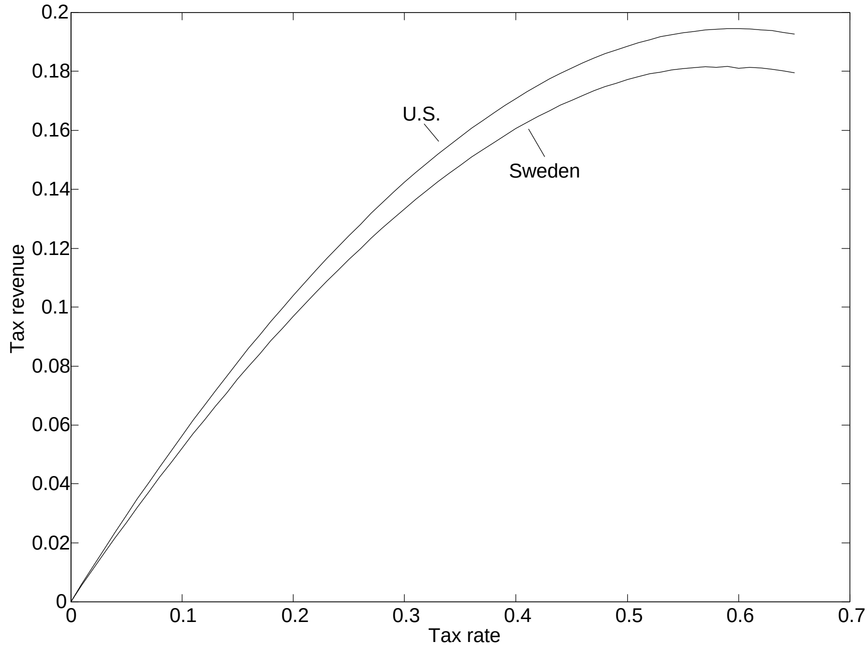

Figure 2: Laffer curves

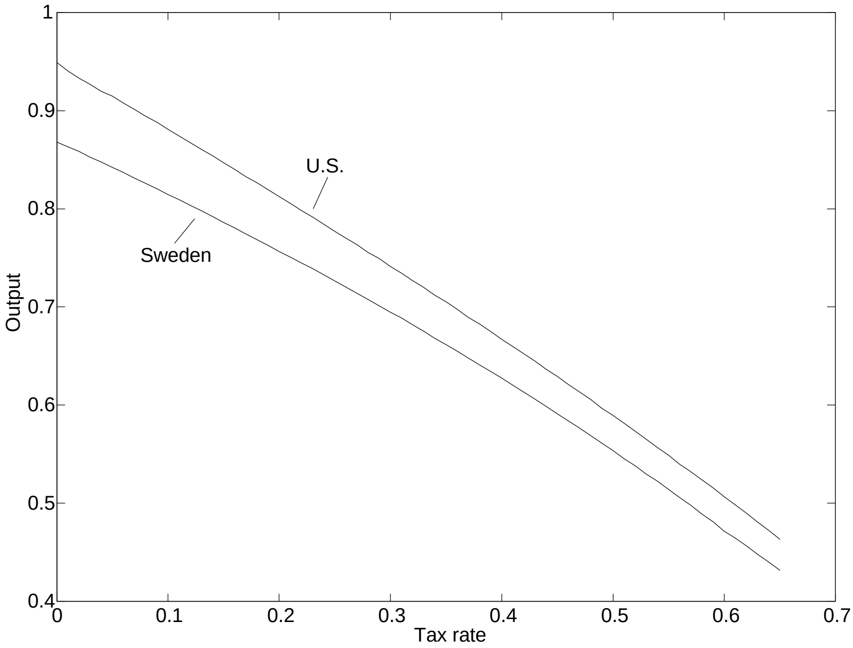

Figure 3: Output per capita

Chapter 4

The Effectiveness of Government Debt and Transfers as Insurance*

1 Introduction

If individuals are borrowing constrained and if private insurance markets are missing, government debt and redistributive taxation can help individuals to smooth consumption when facing idiosyncratic income risk, as Woodford (1990) (for debt), Varian (1980), and Eaton and Rosen (1980) (for taxation) have demonstrated.[1] Recent work has examined the quantitative importance of such policy in plausibly calibrated general equilibrium models. However, the policy instruments were then studied in isolation. In this paper, I allow for both instruments, look for the optimal combination of them, and analyze their effects on economic efficiency, equity, and risk.

The focus here is on insurance against individual specific risks among a large number of heterogenous individuals or households, and I make the simplifying assumption that aggregates are constant. Hence, I examine the choice of average levels of transfers and public debt, but I do not consider how policy should vary over the business cycle or in response to aggregate shocks (see Barro [1979], Lucas

* I thank Torsten Persson and seminar participants at IIES for comments.

[1] See for example Zeldes (1989), Jappelli (1990), and Attanasio and Davis (1996) for evidence on borrowing constraints and imperfect insurance.

and Stokey [1983], and Chari, Christiano, and Kehoe [1994] for such issues).

By issuing debt, the government can increase the liquidity in the economy and facilitate for agents to self insure by private saving. However, financing of interest payments will require distortionary taxation, and government debt might crowd out investment in physical capital. In a recent paper, Aiyagari and McGrattan (1998) considered these trade-offs and found that the optimal level of government debt in the U.S. is close to the actual 67 percent of GDP, but that variations in the size of the debt have small welfare effects.

A more direct way for the government to provide insurance is redistributive taxation which transfers resources to households with low income. Flodén and Lindé (1998) found that the welfare implications of varying the level of government transfers and taxes can be large. In a model calibrated to the U.S. economy, the optimal size of the insurance program was found to be 18 percent of output, and people would be prepared to give up more than five percent of annual consumption to get this program instead of one with no transfers at all.

The main difference, in addition to examining different policy instruments, between Aiyagari and McGrattan's paper and Flodén and Lindé's paper is the amount of uncertainty agents are assumed to face. The wage process estimated by Flodén and Lindé is considerably more persistent than the one used in Aiyagari and McGrattan. When the wage process is persistent, agents need to accumulate large buffer stocks to insure against bad times. Hence, as persistence increases, the welfare costs of missing insurance markets will increase, and so will the welfare gains of optimal government policy. I find, however, that the small welfare effects in Aiyagari and McGrattan's paper were not only a result of the low persistence in the wage process but also a result of government debt being a weak insurance instrument.

The government policies considered have redistributive effects in addition to the insurance effects. For example, proportional taxes in combination with lump sum transfers shift resources from high- to low-income households. With the commonly used utilitarian welfare function, both reduction in risk and increased equity will raise welfare. This measure sums the utility of all agents. Since utility functions are concave, taking from a wealthy consumer and giving to a poor will increase the sum of utilities, holding prices and decision rules fixed. Bénabou (1998) has suggested a welfare measure which only includes insurance effects. The idea is to calculate certainty equivalent consumption bundles for each individual and evaluate the utility of an agent who consumes the average of these certainty equivalent bundles. A reduction in uncertainty will increase certainty equivalent consumption for all individuals affected, while a pure redistribution of resources will reduce the certainty equivalent consumption for one agent and increase the consumption for an other agent by (at least approximately) the same amount. This measure is useful for separating insurance effects from redistribution effects, but I see no natural interpretation of it as a measure of economy-wide welfare.

The results confirm the main findings in the previous studies cited, namely that the welfare gains of shifting to the optimal tax and transfer program can be significant while welfare effects of changes in debt levels are small. In this framework, the utilitarian welfare function is maximized when transfers are 22 percent of output and debt is -75 percent of output. Transfers dominate debt as a policy instrument – when more insurance is provided from the tax system, the optimal debt level falls.

Bénabou's welfare measure shows that increases in public debt significantly can improve insurance possibilities if transfers are lower than what is optimal. Not surprisingly, however, I also find that increases in debt have negative equity effects and hence mainly benefit the wealthy consumers. They benefit from the higher return on capital implied by the increased debt while all agents suffer from the reduction in output (and thus also in wages and transfers) which is a consequence of higher taxes and crowding-out of physical capital. Thus, Aiyagari and McGrattan's finding that the optimal level of public debt is 67 percent of output does no longer hold when the government also can choose the level of transfers optimally.

In Section 2, I present a model similar to that in Aiyagari and McGrattan (1998). Then, in Section 3, I first examine what the optimal level of transfers is if debt is held fixed at its present level, and what the optimal level of debt is if transfers are held fixed at the present level. Thereafter I do the main experiment of allowing for both instruments simultaneously. In addition to looking for optimal policies, I examine how the instruments operate. I study the effects on efficiency, equity, and insurance. Section 4 concludes.

2 The model

There are a large number of firms and infinitely lived agents, and these take prices as given. Firms behave competitively and use capital and labor to produce a single commodity. There is no aggregate uncertainty, and no uncertainty faced by firms.

Agents get income by renting capital to firms at a certain rate, and from labor income which depend on their individual specific productivity. Individuals are born with permanently different abilities, and in addition productivity varies over time. Agents are borrowing constrained (that is, net financial wealth is not allowed to be negative) and there is no market for private insurance.[2] To smooth consumption over time, and to self insure against future income risk, agents therefore accumulate buffer stocks of the safe asset in good times.

There is also a government, which can issue public debt and tax labor and capital income. Tax revenues are spent on public consumption, transfers to households, and interest payments on the debt.

In equilibrium, the price of capital must be such that agents are willing to

[2] These market imperfections are imposed exogenously. A growing literature (see Atkeson and Lucas [1995] and Krueger and Perri [1998] for recent contributions) studies the feasibility of private insurance markets under asymmetric information. The paper by Cole and Kocherlakota (1998) supports the view that a limited number of financial assets (e.g. one risk-free bond) may provide the best feasible insurance.

hold precisely the amount of capital demanded by firms. For example, if wage uncertainty increases, agents will try to accumulate more precautionary savings and consequently the equilibrium after-tax interest rate must fall so that firms are willing to engage the capital in their production. Or, if public debt is increased, the interest rate must increase (and consequently the real capital stock must shrink) so that agents are prepared to hold the additional amount of bonds.

Let us now take a closer look at the model.

2.1 Production and factor prices

There is a continuum of firms which have Cobb-Douglas production functions and behave competitively in product and factor markets. Let K denote the aggregate capital stock and H the aggregate labor supply in efficiency units. The economy grows at the exogenous rate g and there is no aggregate uncertainty in the economy. Aggregate production is then given by

$$Y_t = F_t\left(K_t, H_t\right) = K_t^{\theta}\left[\left(1+g\right)^t H_t\right]^{1-\theta}.$$

Factor market clearance requires that the interest rate is

$$r_t = F_{t,K}\left(K_t, H_t\right) - \delta,$$

where δ denotes the depreciation rate of capital, and that the wage rate is

$$w_t = F_{t,H}\left(K, H\right).$$

Only balanced growth paths where aggregate output, capital and consumption grow at rate g while aggregate labor supply is constant will be considered. Hence the interest rate will be constant. It will often be convenient to look at detrended variables. Define therefore

$$\tilde{w} \equiv w_t / Y_t,$$

which also is constant in equilibrium.

2.2 The government

The government budget constraint is

$$G_t + B_t + rD_t = D_{t+1} - D_t + T_t,$$

where G_t is government consumptions, B_t are lump sum transfers to agents, D_t is the government debt, and T_t is tax revenue. Government consumption is exogenous and a constant fraction of GDP, $\bar{G} = G_t/Y_t$.

Tax revenues are levied by taxing capital and labor at the common flat rate τ. After-tax factor prices are defined as $\bar{r} = (1-\tau)\,r$, and $\bar{w} = (1-\tau)\,\tilde{w}$.

The levels of debt and transfers are choice variables for the government. These will be chosen to maximize some social welfare function in the stationary balanced growth equilibrium. That is, welfare effects accrued during the transition from the current economy to the new equilibrium are ignored.

2.3 Households

The model is populated by a continuum of infinitely lived, ex ante identical, agents of unit mass. Let s_t denote an agent's productivity state at time t. The productivity process is a finite state Markov chain, $s \in \mathbf{S}$ with transition probabilities $\Gamma(s, s')$ denoting the probability of transition from productivity state s today to s' tomorrow. Moreover, let $q(s_t) \equiv q_t$ be the productivity associated with state s_t.

Each agent is endowed with one unit of time, which is divided between labor, h, and leisure, l. Income can be consumed or saved in a safe asset. Let b_t denote a lump sum transfer from the government, a_t the agent's asset holdings at the end of period $t-1$, and c_t consumption. Define then $\bar{b} = b_t/Y_t$, $\tilde{a}_t = a_t/Y_t$, and $\tilde{c}_t = c_t/Y_t$. The budget constraint for an agent at time t is thus

$$(1+g)\,\tilde{a}_{t+1} = \bar{b} + \bar{w}q_t h_t + (1+\bar{r})\,\tilde{a}_t - \tilde{c}_t. \tag{4.1}$$

Agents are also subject to a no-borrowing constraint,

$$\tilde{a}_t \geq 0. \tag{4.2}$$

The asset holdings, $\tilde{a}$, are restricted to belong to $\mathbf{A} = [0, \bar{a}]$ where $\bar{a}$ is chosen high enough never to be a binding condition.

The instantaneous utility function is,

$$u(c,h) = \frac{c^{1-\mu} \exp\left[-(1-\mu)\zeta h^{1+\gamma}\right]}{1-\mu},$$

where ζ is a constant, μ is the risk aversion for consumption fluctuations, and $1/\gamma$ is the labor-supply elasticity. This utility function is consistent with aggregate hours worked being constant in a growing economy (see King, Plosser, and Rebelo, 1988). We can rewrite the utility as

$$u_t(c,h) = Y_0^{1-\mu}(1+g)^{t(1-\mu)} u(\tilde{c},h)$$

Agents maximize their expected life-time utility,

$$U_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where β is the time discount rate. The detrended formulation of the objective is

$$U_0 = Y_0^{1-\mu} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (1+g)^{t(1-\mu)} u(\tilde{c}_t, h_t).$$

The normalizations allow us to formalize a time-independent problem. With primes denoting next period's variables, the Bellman equation is

$$v(\tilde{a}, s) = \max_{\{\tilde{a}', h\}} Y_0^{1-\mu} u(\tilde{c}, h) + \beta (1+g)^{1-\mu} \sum_{\mathbf{S}} \left[v(\tilde{a}', s') \Gamma(s, s')\right] \tag{4.3}$$

subject to (4.1), (4.2), and $0 \leq h \leq 1$.

2.4 Equilibrium

A stationary equilibrium is given by (i) a constant tax rate τ, constant government debt, $\bar{D} \equiv D/Y$, and constant transfers, $\bar{B} \equiv B/Y$, (ii) a constant interest rate $\bar{r}$ and wage rate $\bar{w}$, (iii) time invariant decision rules for agents' asset holdings, $\tilde{a}' = a(\tilde{a}, s)$, and hours worked, $h = h(\tilde{a}, s)$, (iv) a measure of agents over the state space, $\lambda(\tilde{a}, s)$, (v) aggregate values for asset holdings, $\bar{A} = \int a(\tilde{a}, s)\, d\lambda$, and for the number of efficiency hours worked, $H = \int q h(\tilde{a}, s)\, d\lambda$, such that the following equilibrium conditions are fulfilled:

- Decision rules solve agents' maximization problem, given by (4.3).

- Tax revenues equal government expenses,

$$\tau \int_{\mathbf{A}\times\mathbf{S}} \left[\tilde{w}qh\left(\tilde{a},s\right) + ra\left(\tilde{a},s\right)\right] d\lambda = \bar{B} + \left(r-g\right)\bar{D} + \bar{G}.$$

- Factor markets clear.

- Aggregate savings are equal to firms' demand for capital plus government debt,

$$\bar{A} = \bar{K} + \bar{D},$$

  where $\bar{K} \equiv K/Y$.

- The measure of agents over the state space is invariant, i.e.

$$\lambda\left(\mathbf{a},\mathbf{s}\right) = \int_{\mathbf{A}\times\mathbf{S}} P\left(a,q,\mathbf{a},\mathbf{s}\right) d\lambda,$$

  for all $\mathbf{a}\times\mathbf{s} \subseteq \mathbf{A}\times\mathbf{S}$. The transition function P is the probability that an agent with state (a,s) will have a state belonging to $\mathbf{a}\times\mathbf{s}$ next period,

$$P\left(\tilde{a},s,\mathbf{a},\mathbf{s}\right) = \sum_{s'\in\mathbf{s}} \mathcal{I}\left[a\left(\tilde{a},s\right)\in\mathbf{a}\right]\Gamma\left(s,s'\right),$$

  where $\mathcal{I}$ is an indicator function.

2.5 Computation of equilibrium

To find the agents' decision rules for saving and labor supply, I discretize the state space and make a piecewise linear approximation of the agents' decision rules over this.[3] To solve for the equilibrium, I use an algorithm inspired by Huggett (1993) and Aiyagari (1994). The algorithm consists of the following steps: Fix the transfer and debt, and guess an interest rate, r, and the average

[3] The state space is approximated by a grid consisting of 70 values for asset holdings, one high and one low value for the permanent shock, and 7 values for the temporary wage level. The Markov chain for wages is approximated with the algorithm by Tauchen (1986). I use a spread of $\pm 3\sigma_\varepsilon/\left(1-\rho^2\right)^{1/2}$ for the wage grid. The step size in the grid for asset holdings is increasing in wealth.

efficiency hours of labor supply, $\hat{H}$. Then solve for the wage per efficiency unit of labor as a function of r and $\hat{H}$, and calculate the tax rate implied by government budget balance, by setting

$$\tau = \frac{\bar{B} + (r - g)\,\bar{D} + \bar{G}}{\tilde{w}\hat{H} + r\left(\bar{K} + \bar{D}\right)}.$$

The agents' decision rules are then solved for, and from those rules, the measure of agents over the state space is computed. Once we have found the measure λ, aggregates such as savings and labor supply are straight forward to calculate. If the implied aggregate saving of agents does not equal demand for capital, or if the implied labor supply is different than the guess, then make new guesses and start over. If both equalities hold, the equilibrium of the economy with tax rate τ and debt level $\bar{D}$ has been found.

2.6 Parameterization

With two exceptions, Aiyagari and McGrattan's (1998) parameter values are used. One model period thus corresponds to one year, and I set risk aversion to 1.5, government consumption to 21.7 percent of GDP, the capital share of income to 0.30, the depreciation rate of capital to 0.075, and the growth rate of GDP to 1.85 percent. The discount factor β is set to 0.9884 to get an interest rate close to 4.5 percent as in U.S. data.

The two deviations from Aiyagari and McGrattan's parameter choices are in the utility function and the productivity process. Estimations of the wage elasticity of labor supply vary widely between studies. However, most estimates of the elasticity are less than 0.5 for men and the estimated elasticity for women is typically higher than that for men – see for example MaCurdy (1981) and Altonji (1986). I set the elasticity to 0.5 which is considerably lower than the value used by Aiyagari and McGrattan. The fraction of available time an agent devotes to labor is determined by ζ, which is set so that average labor supply is approximately 0.3 in the benchmark economy.

The productivity process is calibrated as follows. Wages consist of two components, one permanent ability level, ψ, and one temporary component, z_t. More specifically, an agent's labor productivity as a fraction of the average productivity at time t is $q_t = e^{\psi + z_t}$. Across individuals, ψ is *iid* and has mean zero and variance σ_ψ^2. Productivity evolves stochastically over time according to the process

$$z_t = \rho z_{t-1} + \varepsilon_t,$$

where ρ determines the degree of persistence of shocks, and ε is *iid* normally distributed with mean zero and variance σ_ε^2.

Aiyagari and McGrattan set $\rho = 0.60$ and $\sigma_\varepsilon = 0.24$. Recent evidence, for example Card (1991), Storesletten, Telmer, and Yaron (1997), and Flodén and Lindé (1998), suggest that wage and income processes are more persistent than that. The values $\rho = 0.90$ and $\sigma_\varepsilon = 0.21$, are in line with estimates in these papers, and this wage process is used as the benchmark here. Both Card's and Flodén and Lindé's estimates of the standard deviation of permanent wage differences, σ_ψ, suggest that it is approximately 0.34.

The parameter values for the benchmark calibration of the model are then $\mu = 1.5$, $\gamma = 2$, $\zeta = 9.1449$, $\beta = 0.9884$, $g = 0.0185$, $\theta = 0.3$, $\delta = 0.075$, $\bar{G} = 0.217$, $\rho = 0.90$, $\sigma_\varepsilon = 0.21$, and $\sigma_\psi = 0.34$.

3 Welfare effects of debt and transfers

Two welfare measures are used to evaluate the implications of government policy. The first measure is the average utility of agents, or equivalently in this balanced growth setting, the average of agents' value functions at any point in time, for example

$$W^U = \int u_0 \left(c \left(\tilde{a}, q \right), h \left(\tilde{a}, q \right) \right) d\lambda$$

This is the utilitarian approach, which was used in Aiyagari and McGrattan (1998) and Flodén and Lindé (1998). The utilitarian welfare function includes both insurance and redistribution effects. Holding prices and decision rules fixed, reductions in uncertainty and redistribution from rich to poor will enhance welfare by this measure.

The other measure follows Bénabou (1998) and allows us to isolate the welfare gains or losses of insurance from those of redistribution. Consider an agent facing the uncertain stream of consumption and leisure $\{c_{t+j}, l_{t+j}\}_{j=0}^{\infty}$. The general approach is then to find a certain consumption-leisure stream, $\left\{\bar{c}_{t+j}, \bar{l}_{t+j}\right\}_{j=0}^{\infty}$, which renders the same expected utility as the uncertain stream $\{c_{t+j}, l_{t+j}\}_{j=0}^{\infty}$. A restriction we want to impose on the certainty equivalent consumption-leisure bundle is that aggregates remain stationary, that is $\int \bar{c}_{t+j} d\lambda_t = \bar{C}$, $\int \bar{l}_{t+j} d\lambda_t = \bar{L}$ for all $j \geq 0$. When we have found a certainty equivalent consumption-leisure bundle, the measure of social welfare is computed as the utility of a representative agent who consumes the certainty equivalent aggregates $\bar{C}$ and $\bar{L}$,

$$W^B = u_0 \left(\bar{C}, 1 - \bar{L}\right)$$

This welfare measure captures insurance effects only. A reduction in uncertainty will increase the certainty equivalent consumption and/or leisure and hence increase welfare. But pure redistribution, holding prices and decisions fixed, will leave the measure unaffected.

In Bénabou's problem, consumption was (in practice) the only decision variable, and the natural choice of certainty equivalent consumption was a constant value for $\bar{c}$. Here, however, there are several ways to calculate certainty equivalent consumption-leisure streams. I compute two certainty equivalent bundles. In one, I restrict each agent's leisure to be fixed at the average leisure level in the economy. The other bundle is calculated with leisure fixed at the level which the agent chose at time t under uncertainty. In both measures, then, I look for the $\bar{c}$ which makes the value of $\left\{\bar{c}, \bar{l}\right\}_t^{\infty}$ equal to the expected value of $\{c_{t+j}, l_{t+j}\}_{j=0}^{\infty}$.

The implications of fixing leisure at the average level or at the initial individual choice level are somewhat different since there is not a one-to-one mapping between the instantaneous and life time utility. For example, an agent with little wealth but high labor productivity may work hard today and not enjoy the rewards for this until later. It is unclear how such effects aggregate into the welfare measure under the different assumptions. The differences between the two welfare measures turn out to be quantitatively small, and I only report results for leisure fixed at the choice level.

3.1 The optimal quantity of debt and transfers revisited

Holding government debt fixed at 2/3 of GDP, what is the optimal level of transfers? The answer obviously depends on which welfare measure we use. In Figure 1, I plot the welfare gain compared to the benchmark economy as a function of the transfer level. This is done for the utilitarian welfare measure, denoted U, which includes both insurance and redistribution effects, and for Bénabou's measure, denoted I. The welfare gain is quantified with a consumption compensating variation measure, i.e. the percentage decrease in annual consumption needed for agents in the economy under consideration to get the same utility (in terms of the chosen welfare measure) as in the benchmark economy.

This exercise confirms Flodén and Lindé's (1998) finding that the optimal level of transfers is high, and that welfare gains of optimal policy can be significant. The optimal level of transfers is 19 percent of GDP with the utilitarian welfare measure, and the welfare gain of having this level instead of the benchmark (8.2 percent) is 2.5 percent of annual consumption.

As expected, a part of the result is due to redistribution effects. As transfers and taxes increase, resources are shifted from high- to low-income households, and because of the concavity of the utility functions, this redistribution will tend to increase the average utility in the economy. However, Figure 1 also shows that there are pure insurance benefits of government transfer systems. The welfare measure I peaks when transfers are 13 percent of output.

Figure 2 shows that the insurance effects of government debt are fairly strong, but that increases in debt have negative redistribution effects. Holding transfers fixed at 8.2 percent, the utilitarian welfare function is maximized when debt is 150 percent of output while Bénabou's welfare function is maximized when debt is more than 200 percent of GDP. This implies that government debt primarily helps wealthy and high-income households to smooth consumption, an issue I will return to below.

The findings here are somewhat different from Aiyagari and McGrattan's (1998) findings – the optimal debt level is higher, and the welfare effects of changing the debt level are larger. Two differences in the setups are underlying this discrepancy. First, the labor-supply elasticity is lower in the current paper, implying that distortions on labor supply are smaller. Second, there is considerably more idiosyncratic risk and heterogeneity in the current paper. There is therefore a larger demand for insurance and more scope for redistribution.

3.2 The optimal combination of debt and transfers

The optimal combination of transfers and debt is the pair, $\left(\bar{B}, \bar{D}\right) \in \mathbf{B} \times \mathbf{D}$, which maximizes the social welfare function. The transfer and debt levels considered are $\mathbf{B} = \{0.00, 0.01, ..., 0.25\}$, and $\mathbf{D} = \{-1.00, -0.75, ..., 2.00\}$. Figure 5 shows results for some of the (B, D) combinations. The utilitarian welfare function is maximized when transfers are 22 percent of output and debt -0.75 percent of output. When agents are provided insurance from the tax system, all the benefits from public debt vanishes. If the social planner only cares about the insurance value of the policy, the optimal transfer level is around 10 percent and the optimal size of public debt is high.

Table 1 shows some selected statistics for the U.S. economy, for the benchmark economy, and for the economy that is optimal according to the model. The economy with the optimal policy does not look drastically different from the benchmark economy. Consumption is smoother while hours worked are somewhat more volatile.

The choice of wage process is critical for the amount of risk agents face, and the degree of persistence is particularly important. The unconditional variance for the AR(1) process is $\sigma_\varepsilon^2/(1-\rho^2)$, which is sensitive to changes in persistence when ρ is close to unity. There is indeed controversy over what the value of ρ is, and Aiyagari and McGrattan (1998), together with a number of other papers, claim that $\rho = 0.60$ and $\sigma_\varepsilon = 0.24$. I have solved the model with this parameterization also (but keeping the permanent wage differences, $\sigma_\psi = 0.34$). The optimal combination policy combination is then $B = 0.16$ and $D = -1.00$, and the welfare gain over the benchmark economy is 0.7 percent of annual consumption.

3.3 Effects on inequality and distributions

From the top-left graphs in Figures 1 and 2, and from Figure 5, we see that both debt and transfers can have large (positive) insurance effects. However, the ways in which these two instruments work seem to be very different. As debt is increased, more insurance is added to the economy. We see in the graphs that the welfare measure including only insurance effects peaks at considerably higher debt levels than the utilitarian welfare measure. That is, increases in debt mostly benefit the wealthy individuals and effects on equity are negative. Figure 4, though, shows that asset and income distributions do not change much when debt is increased from 67 to 150 percent of output. The most drastic change is a clear reduction in the number of agents with no wealth. People more willingly engage in precautionary savings when the return on capital increases, and it is consequently less likely that their buffer stocks are depleted. The gini coefficients reported in Figure 6 further confirm that income and consumption distributions do not change much when debt is changed.

We also see that the asset distribution becomes more compressed when debt increases, something which could indicate that debt has positive redistribution effects. This would be an incorrect conclusion, though. When debt is increased, taxes must (in most cases) be increased and real capital is crowded out. Hence the level of production decreases as is seen in Figure 2. Although fewer agents are in the worst state of the world, i.e. have low earnings and no wealth, those who are there will suffer badly from any reduction in the level of production since then transfers and wages are reduced in proportion. Because the marginal utility of these agents is high, they will be important in the utilitarian maximization problem.

The distributional effects of changes in transfers are more obvious. Higher benefit levels reduce uncertainty since the lowest income level improves relative to the highest income level. Higher benefits also imply less production since increased taxes discourage work effort and saving, and since reduced uncertainty reduces the need for precautionary saving. In addition to providing insurance, higher transfer levels make the tax system more redistributive. It is then not surprising that, as we see in Figures 1 and 5, utilitarian welfare measure peaks at higher transfer levels than the insurance-only measure.

The model have clear predictions for how variations in transfers will affect distributions. As transfers increase, asset and earning distributions become more dispersed while the distributions of consumption and gross income become more compressed. Wealth gets more dispersed since more agents choose to hold no wealth at all when transfers are high. People with low wages do no longer have a desperate need to save in case times get even worse. With higher benefit levels, there are also less incentives for people with low wages and modest wealth to work.

Table 2 contains some additional information on the distributional effects of policy. I have conducted the following experiment. Assume that a worker observes his current productivity state (ψ, z). Conditional on this information, if the worker could choose a policy (B, D) and immediately move to the steady state in that economy, which policy would he (or she) choose? I have restricted the policy space to $(B, D) \in \{0.00, 0.05, ..., 0.30\} \times \{-0.50, -0.25, ..., 2.50\}$. In the approximation of the productivity space, there is one low and one high value for permanent effects, ψ, and seven values for the temporary productivity, z. We see in the table that almost all agents with low permanent productivity choose the maximum level of transfers and the minimum level of debt while the opposite holds for those with high permanent productivity.

The above exercise also hints at some interesting political economy aspects. If the 'optimal' level of transfers is 22 percent of output, why is that level not implemented in reality? One answer could be that the political process does not implement the utilitarian welfare function. Table 2 indicates that the median voter would in fact choose a transfer level around 10 percent of output, close to the actual U.S. level.[4]

4 Summary and conclusion

This study confirms Aiyagari and McGrattan's finding that, with a utilitarian welfare measure, changes in government debt have small welfare implications. However, I find that the *insurance* value of increasing debt can be large, but that this effect is offset by negative effects on equity. Welfare gains of increasing transfer levels can be large, both because of additions to insurance and to equity.

In providing insurance and redistribution, transfers will work more efficiently than public debt. In fact, when the government is allowed to choose transfers optimally, the role for public debt vanishes and the optimal level of debt is -75 percent of output.

According to the model, the welfare gains of shifting from today's level of debt (67 percent of output) and transfers (8.2 percent) to the optimal levels, -75 and 22 percent respectively, would be 3.2 percent of annual consumption. This analysis abstracts from the welfare benefits or losses associated with the transition to the new steady state. The capital stock will be decreasing in the transition from a steady state with low transfers to one with high transfers. There are then extra welfare gains from the transition, since part of the capital stock can be consumed in the transition. On the other hand, reducing debt is costly

[4] See Aiyagari and Peled (1995) for a similar exercise.

during the transition. In that transition, debt must be repaid and additional investments in physical capital are needed.

If reducing the public debt from today's level is not politically feasible, the optimal transfer level is 19 percent of output and the welfare gain over current policy is 2.5 percent of annual consumption.

There are many ways in which tax and transfer schemes can be set up. Here, I only allowed for a flat tax rate on labor income, and lump sum transfers. It would also be interesting to examine the potential benefits of targeting transfers to low-income workers or allowing for more progressive tax systems. Directed transfers and more progressivity would add to the distortions in the economy, but also enhance the insurance effects.

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Table 1. Properties of U.S. and Model Economies

Statistic	*U.S.*	*Benchmark*	*Optimal*
interest rate, pre-tax	4.50	*4.48*	5.55
interest rate, post-tax	n/a	2.80	2.66
capital-output ratio	2.50	*2.50*	2.30
hours	.33	*.29*	.26
tax rate	.38	*.38*	.52
c.v., hours	.15	.16	.18
c.v., consumption	n/a	.46	.38
c.v., assets	n/a	1.29	1.64
c.v., disp. income	n/a	.73	.58
gini, consumption	n/a	.25	.21
gini, assets	.78	.61	.70
gini, disp. income	.40	.35	.27
gini, earnings	.63	.42	.43

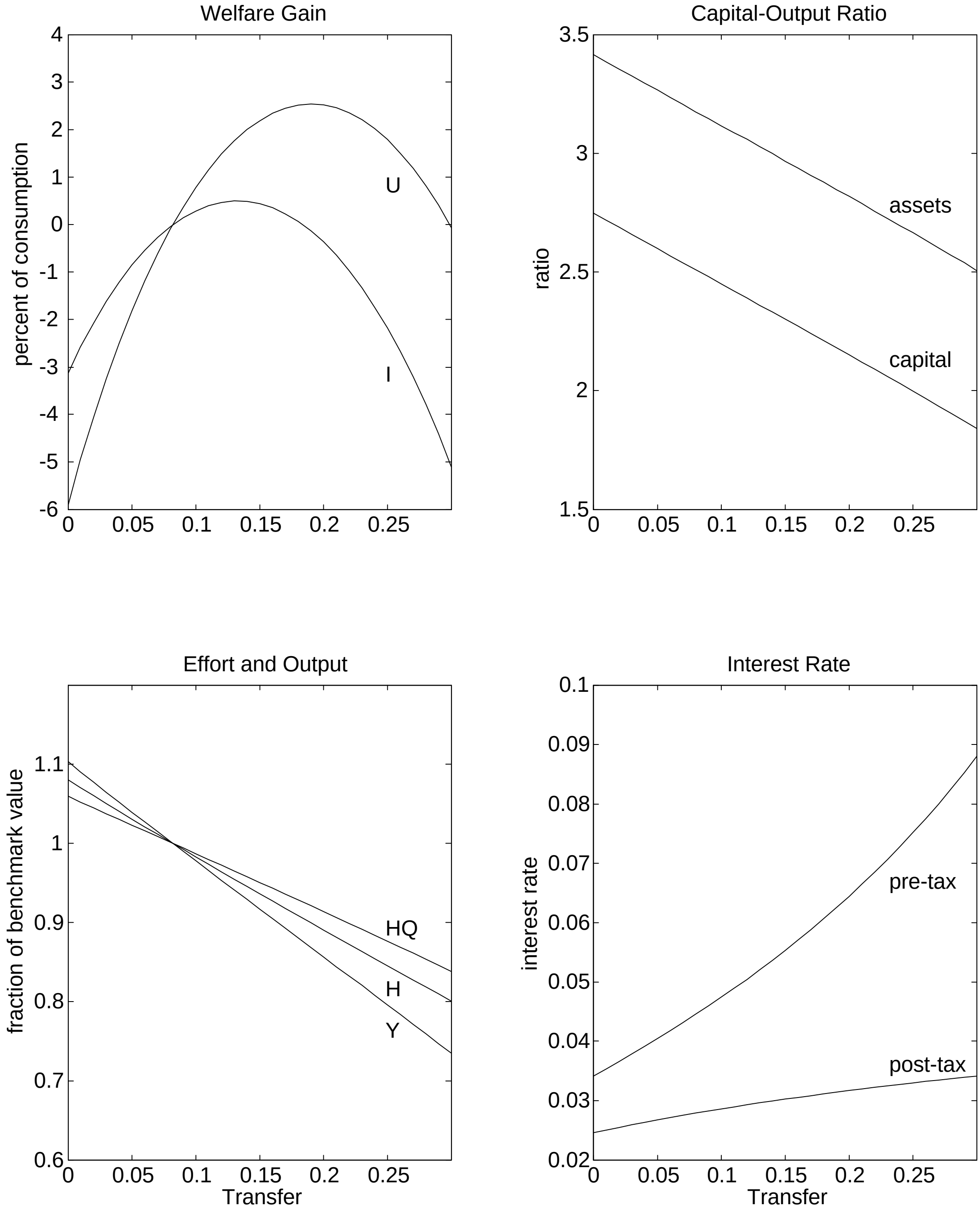
Note: Values in italics have been calibrated to match U.S. data. Coefficients of variation are calculated on the cross-section of individuals. In the benchmark economy, $(B, D) = (.082, .67)$, while $(B, D) = (.22, -.75)$ in the optimal economy. U.S. values are taken from a number of different sources, and some statistics have uncertain values.

Table 2. Optimal Policy Conditional on Idiosyncratic State

	Low ψ			High ψ		
z	w	B^*	D^*	w	B^*	D^*
z_1	.14	.30	$-.50$	.27	.15	2.50
z_2	.22	.30	$-.50$	.43	.10	2.50
z_3	.35	.30	$-.50$	.70	.05	2.50
z_4	.57	.30	$-.50$	1.13	.00	2.50
z_5	.93	.30	$-.50$	1.83	.00	2.50
z_6	1.50	.25	$-.50$	2.97	.00	2.50
z_7	2.44	.10	2.25	4.81	.00	2.50

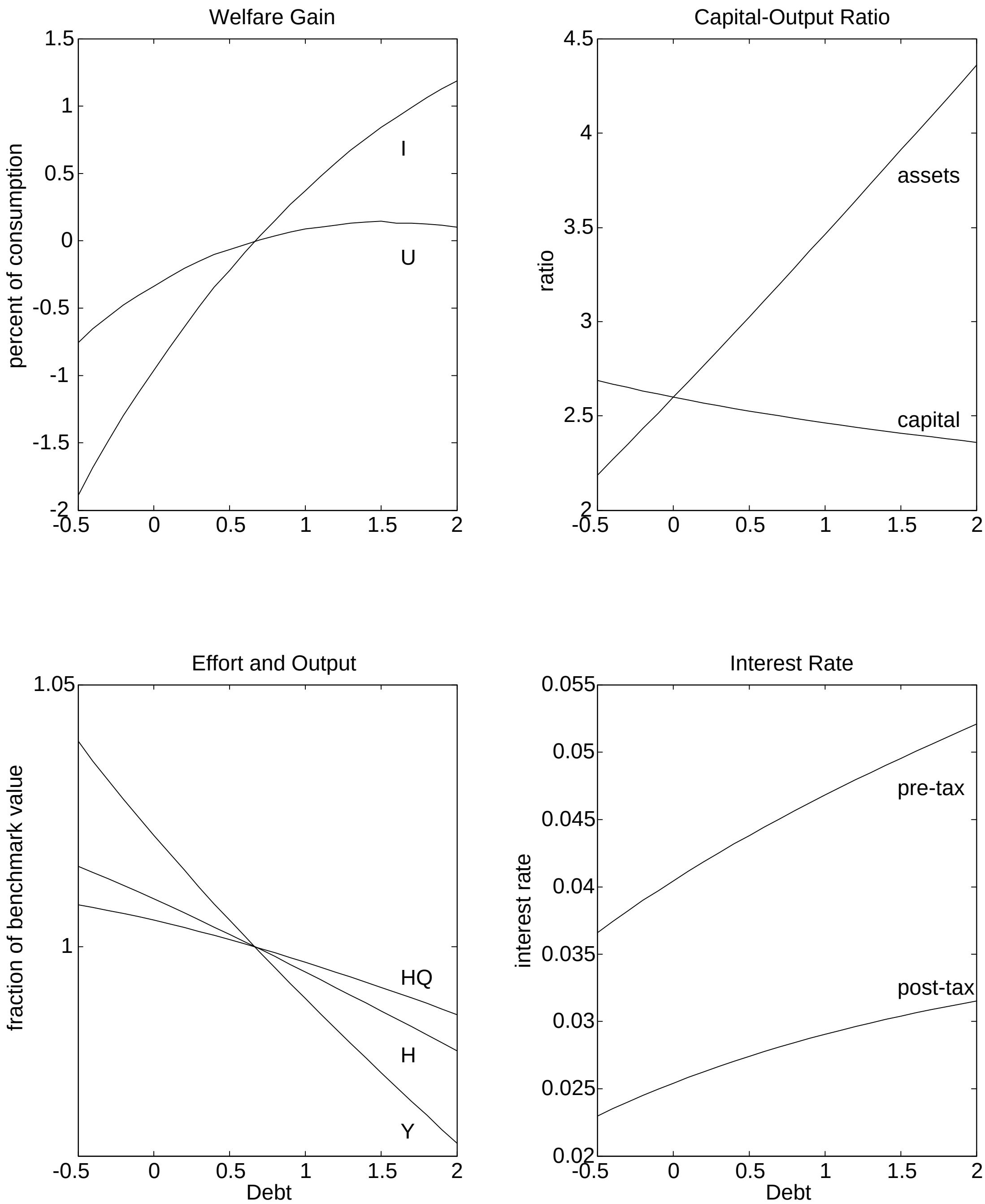
Note: w is the present wage, given by the productivity state (ψ, z), and (B^*, D^*) is the policy (transfers, debt) that the agent would choose if he could make a once-and-forever decision on policy and if the economy would immediately transfer to the new steady state.

Figure 1. Transfer as Only Policy Instrument



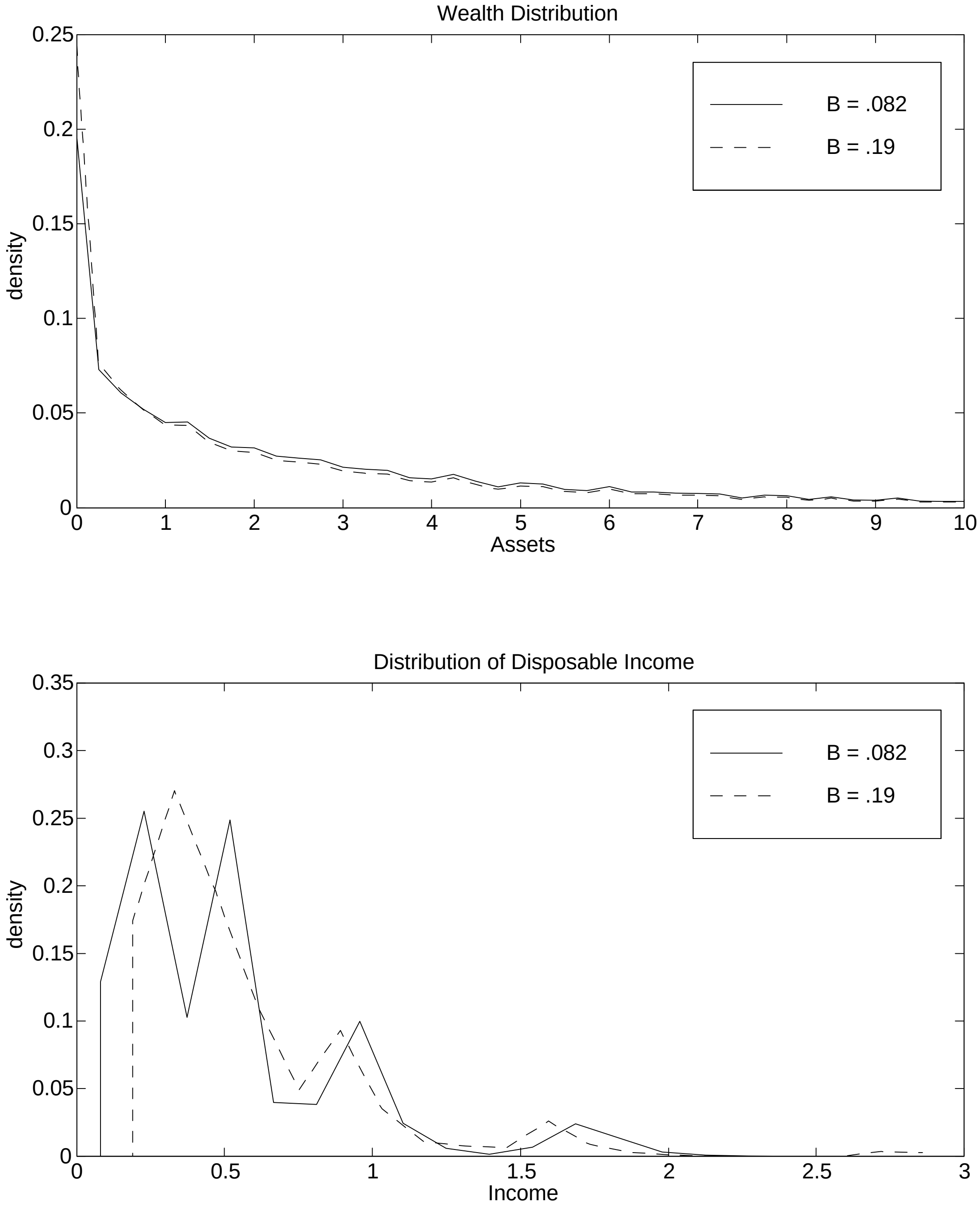
Note: U is the utilitarian utility, including both insurance and redistribution effects. Bénabou's welfare measure, I, only includes insurance effects. H is aggregate labor supply, HQ is aggregate labor supply in efficiency units, Y is aggregate production.

Figure 2. Government Debt as Only Policy Instrument



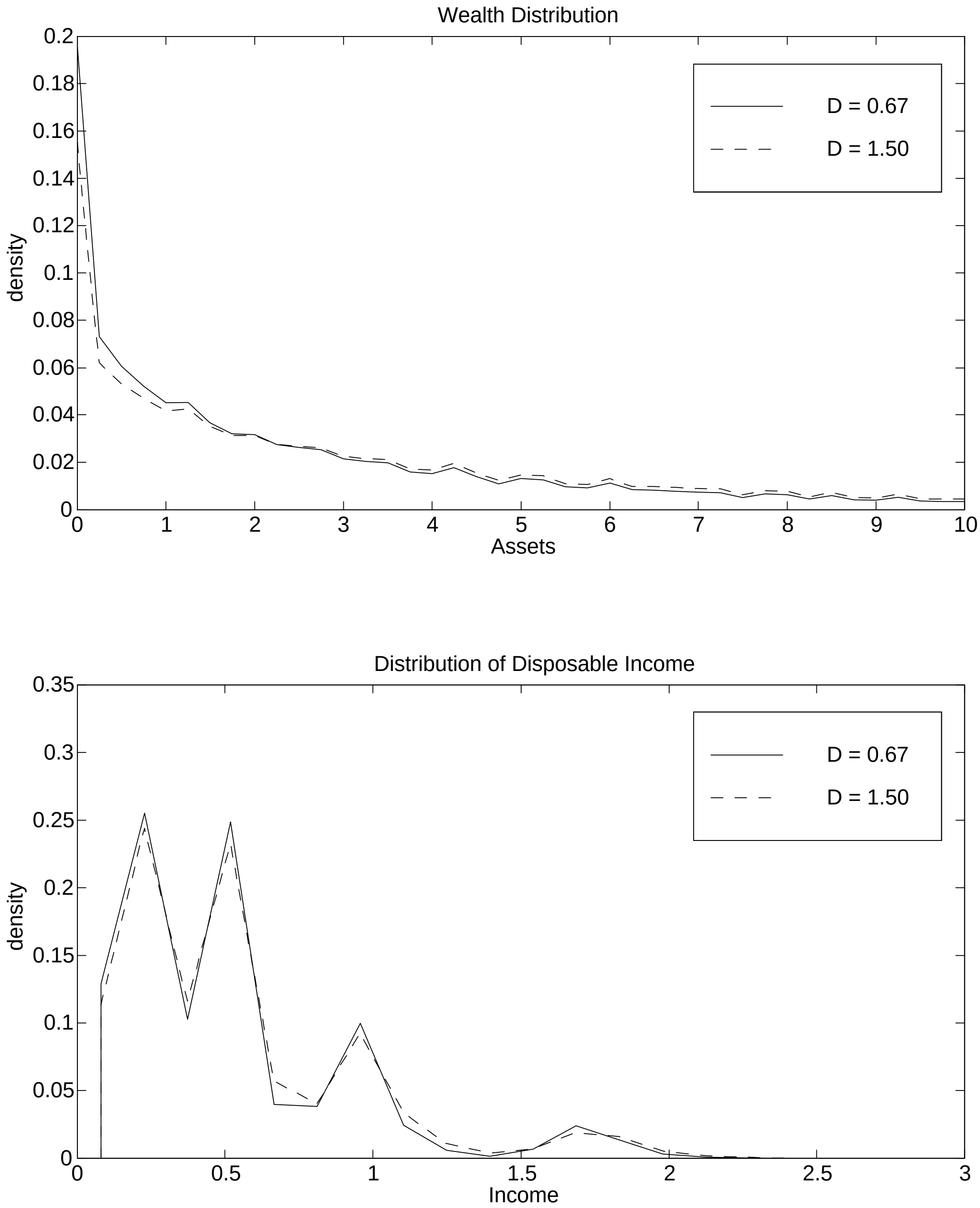
Note: U is the utilitarian utility, including both insurance and redistribution effects. Bénabou's welfare measure, I, only includes insurance effects. H is aggregate labor supply, HQ is aggregate labor supply in efficiency units, Y is aggregate production.

Figure 3. Distribution Effects of Transfers



Note: Assets and income are expressed as fractions of per capita output.

Figure 4. Distribution Effects of Government Debt



Note: Assets and income are expressed as fractions of per capita output.

Figure 5. Debt and Transfers as Policy Instruments

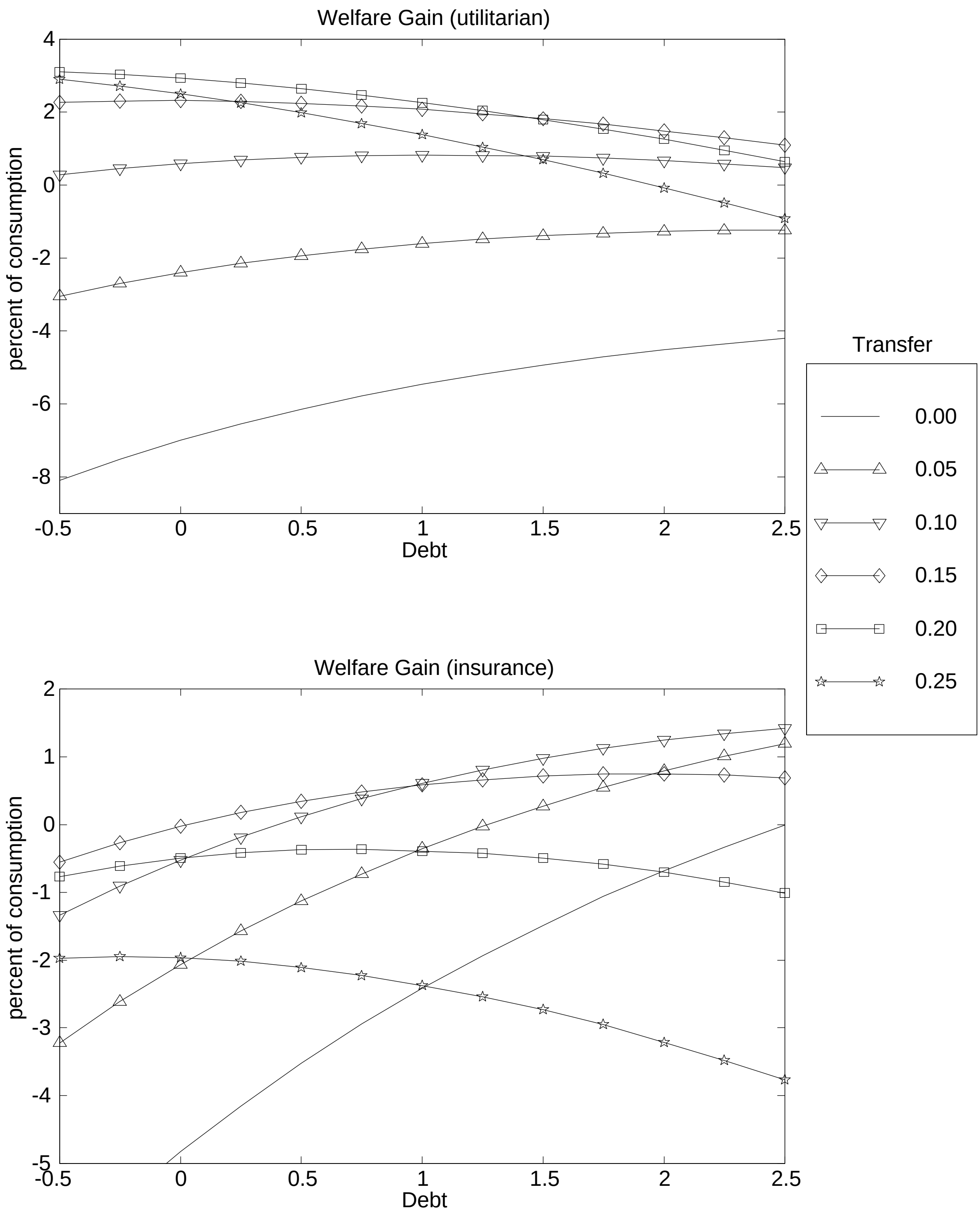
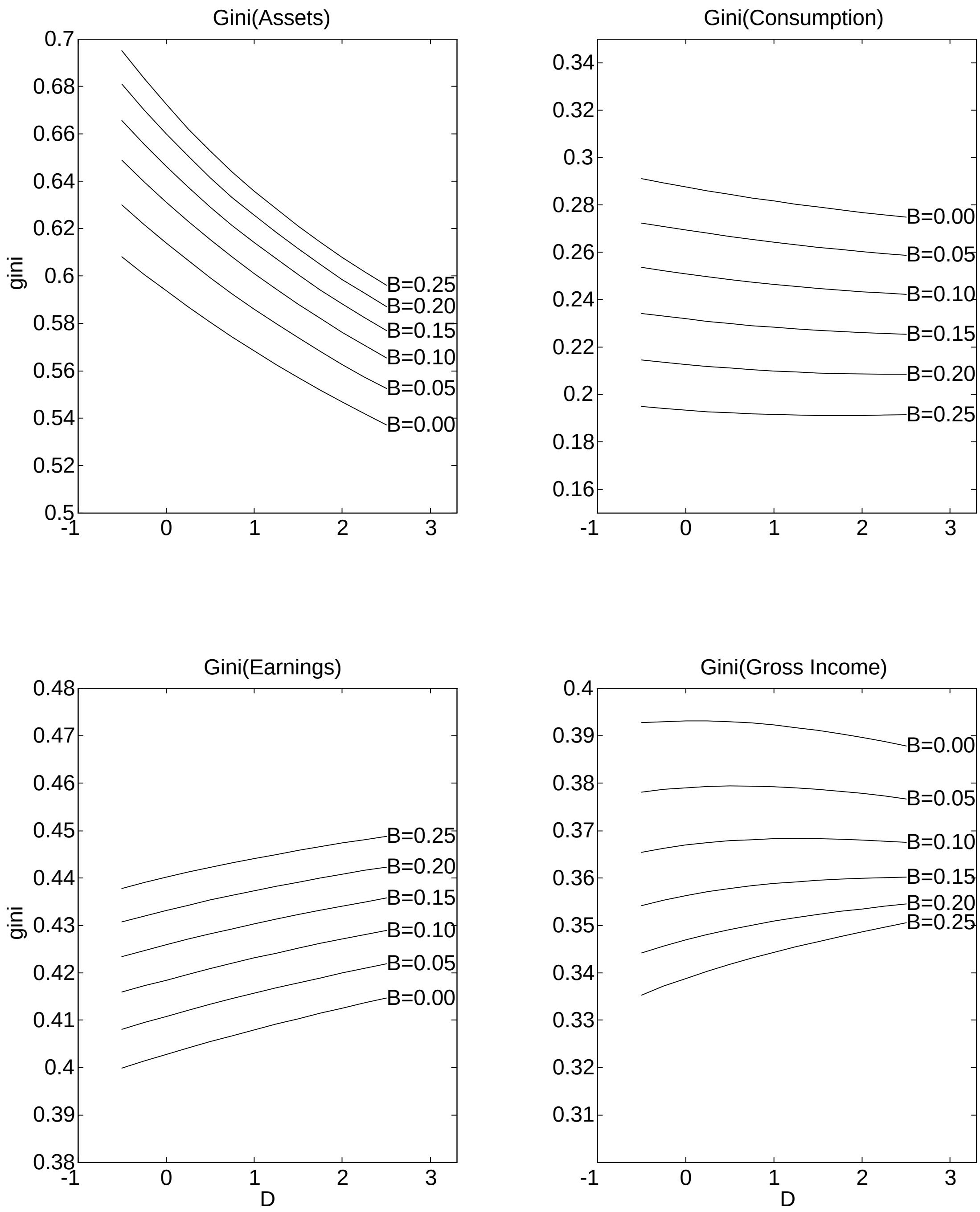


Figure 6. Distribution Effects of Debt and Transfers



Note: Gross income is total income including transfers but before taxes are paid. D is the debt level as a fraction of output and B is the transfer level.

Chapter 5

Endogenous Monetary Policy and the Business Cycle*

1 Introduction

It is widely acknowledged that money, inflation, and output are positively correlated over the business cycle. The behavior of real variables seems to be stable, but there is clear evidence that the relations between real and nominal variables change over time. In a large sample of countries, Backus and Kehoe (1992) find real variables to behave similarly in different subperiods while the behavior of money, inflation, and the price level is changing. Gavin and Kydland (1996) document these facts for U.S. post-war data.

Presumably, variations in the monetary policy is one important explanation to these observations. Even if money does not have any major real effects, changes in money supply certainly have a large impact on nominal variables. If the central bank takes real variables such as output and unemployment into consideration when deciding on money supply, nominal and real variables will be correlated just because of the central bank's reactions to changes in these variables, and if the money supply rule changes, so will correlations between real and nominal

* I thank Finn Kydland, Lars E.O. Svensson, and seminar participants at the Institute for International Economic Studies and Copenhagen University for helpful comments and suggestions.

variables.

In the present paper, the central bank does indeed take the real economy into consideration when deciding on monetary policy. More precisely, I solve for the money supply rule that minimizes the central bank's loss function over inflation and output variability in a dynamic stochastic general equilibrium model. There are shocks both to productivity and in the money supply process. Wages have to be set before contemporaneous shocks and central bank decisions are observed. Hence, unanticipated changes in money supply have real effects.

I find, as did Gavin and Kydland (1996), that changes in the money supply rule can induce large changes in the business cycle behavior of nominal variables. The present paper adds to Gavin and Kydland's analysis by showing that money supply rules can change drastically when central bank preferences change. I find that the quantitative effects that monetary policy has on real variables are small but significant enough to make the optimal money supply rule change a lot when the central bank's weight put on output stability changes. The paper thus shows that sizeable variations in the central bank reaction function can be a reality.

The reason for the instability of the optimal money supply rule is that the central bank faces a trade-off between output and inflation stabilization. When the central bank puts much weight on output stability, its response to a negative productivity shock is as follows. The central bank observes the shock and increases money supply directly. Since nominal wages are assumed to be sticky, this action will decrease real wages and thus stimulate employment. Wage contracts will then be renegotiated, so the central bank cannot exploit the Phillips curve in later periods. Instead, the central bank contracts money supply in successive periods to decrease inflation. This leads to a temporary decrease in the distortionary effects from inflation and stimulates real activity. When, on the other hand, the central bank puts much weight on inflation stability its reactions are different. The central bank does not exploit the Phillips curve at all. Instead it contracts money supply in order to dampen the inflationary tendencies caused by the productivity shock. Compared to the first scenario, the central bank's willingness to use the timing of the inflation tax as an instrument to stabilize output has decreased.

The paper has implication both for empirical and theoretical research on the role of money in the business cycle. When trying to estimate, for example, a vector autoregression including both real and nominal shocks in the system, one must be careful in controlling for changes in monetary regimes. Ideally, one should use short time series for periods of stable monetary policy. Moreover, using high-frequency data (as do for example Bernanke and Mihov, 1995) is an advantage since then, arguably, the central bank cannot influence contemporaneous output. The main implication for theoretical modeling is that we should not expect there to be *one* business cycle behavior of nominal variables, but rather one behavior for each monetary regime.

Before going on to the model and its implications, I will shortly comment on earlier literature in this field. Methodologically, my approach is akin to the real business cycle framework. The model I work with is not purely "real", though, since there are money supply shocks and wage rigidity.

My attempt to introduce money supply in this framework is not new, but until recently a common critique against real business cycle models was their absence or ignorance of monetary issues. Some articles allowed for money, in particular King and Plosser (1984), but the focus was still on the real economy and productivity shocks. Lately, though, several attempts to incorporate effects of monetary policy in dynamic general equilibrium models have been done, for example Cooley and Hansen (1989, 1995) and Huh (1993).[1] Cooley and Hansen (1995) assume that money supply is exogenous and follows an AR(1) process. In reality, however, the central bank reacts to changes in the economic environment when they decide on the monetary policy. This has been captured in the paper by Huh. He postulates a reaction function for the central bank, and this is fitted

[1] Surveys of this literature can be found in Van Els (1995), and Nelson (1997).

to actual data.

In a recent paper, Gavin and Kydland (1996) first document that the volatility and cross-correlations of real variables have been stable in post-war U.S. data but that the correlations between real and nominal variables have changed over time. They then look at a model with a transactions motive for holding money, and experiment with different money supply rules. As expected, they find that changes in the money supply rule have large effects on the correlations between real and nominal variables, but that the behavior of real variables is unaffected by the experiments.

To model monetary policy out of the general equilibrium framework has been typical for research in the real business cycle tradition so far. In this paper, I will assume that the central bank sets monetary policy to minimize a loss function over inflation and output. The central bank is assumed to dislike both inflation in itself and fluctuations in output and inflation. The main difference between my setup and earlier dynamic equilibrium models with money is that money growth was typically fitted to actual data in previous work, whereas I let monetary policy be the equilibrium outcome of the model used. Since the model is only a simplification of the true economy it will be more relevant to relate monetary policy to the model than to data, if we want to learn anything about how monetary policy works and how it (at least theoretically) drives the business cycle. A limitation of the approach is that the central bank's preferences are not derived from the preferences of the agents in the economy.

The paper is organized as follows. In Section 2, I present the model used in the paper. Then, in Section 3, I calibrate the model and look at its business cycle properties. In Section 4, I look at how changes in central bank preferences affect the bank's behavior and the business cycle properties of the simulated economy. Section 5 concludes.

2 Model

The model I use builds on the cash-in-advance model with nominal wage rigidities described in Cooley and Hansen (1995). Here, I extend that model to incorporate endogenous monetary policy. The central bank is assumed to minimize a loss function over output and inflation. I assume the central bank can commit to follow a policy rule which, under some restrictions, is optimal *ex ante*. Since the basic setup of the model is the same as Cooley and Hansen's, I will only give a brief description of it here.

Aggregate production is given by

$$Y_t = e^{z_t} K_t^{\theta} H_t^{1-\theta},$$

where z is the level of productivity, K is the aggregate capital stock and H is the aggregate labor supply. When relevant, lower-case letters denote individual holdings and capital letters denote aggregate holdings.

Productivity is assumed to follow an autoregressive process,

$$z_{t+1} = \rho z_t + \varepsilon_{t+1},$$

where ε is Gaussian white noise.

Letting X denote investment and δ the depreciation rate of physical capital, the capital stock evolves according to

$$K_{t+1} = (1 - \delta) K_t + X_t.$$

There are two consumption goods, c_1 which requires cash, and c_2 which can be bought on credit. Previously accumulated cash balances are thus needed to purchase the "cash good". Purchases of the cash good must then fulfill

$$P_t c_{1t} \leq m_t + (1 + R_{t-1}) b_t + T_t - b_{t+1}, \tag{5.1}$$

where P is the nominal price level, m is money holdings in the beginning of the period, b is bond holdings, R is the nominal interest rate on bonds and T is a lump sum transfer from the government to the households.

Agents also have to fulfill the budget constraint

$$c_{1t} + c_{2t} + x_t + \frac{m_{t+1} + b_{t+1}}{P_t} \leq w_t h_t + r_t k_t + \frac{m_t + (1 + R_{t-1})\, b_t + T_t}{P_t}, \tag{5.2}$$

where w is the real wage rate and r is the real return on capital. These factor returns are determined by the firms' profit maximization and are

$$w_t = (1 - \theta)\, e^{z_t} \left(\frac{K_t}{H_t}\right)^{\theta}$$

and

$$r_t = \theta e^{z_t} \left(\frac{H_t}{K_t}\right)^{1-\theta}.$$

The government's budget constraint is

$$T_t = M_{t+1} - M_t + B_{t+1} - (1 + R_{t-1})\, B_t.$$

To simplify, we assume that the government does not issue any bonds, $B_t \equiv 0$.[2] We then get

$$T_t = M_{t+1} - M_t.$$

Agents have preferences for both consumption goods and for leisure. Each agent's labor input is assumed to be indivisible as in Hansen (1985). The representative agent's utility function is

$$u\,(c_1, c_2, h) = \alpha \ln c_1 + (1 - \alpha) \ln c_2 - \gamma h.$$

Money evolves according to

$$M_{t+1} = e^{\mu_t} M_t \equiv g_t M_t.$$

The money stock is controlled by the central bank, which decides μ_t, perfectly or imperfectly.

[2] The reason for introducing bonds into the model is that they enable us to solve for the nominal interest rate.

2.1 The agents' problem

If nominal interest rates are positive, the cash-in-advance constraint (5.1) will bind. The budget constraint (5.2) will also bind. From these two equations we get

$$Pc_1 = m + M' - M = m + (g-1)M \tag{5.3}$$

and

$$c_1 + c_2 + x + \frac{m'}{P} = wh + rk + \frac{m}{P} + \frac{(g-1)M}{P}. \tag{5.4}$$

Time subindices have been suppressed to simplify notation. Primes denote next period's variables. The money stock and the price level are non-stationary variables. We introduce two new stationary transformations of them,

$$\hat{m} \equiv \frac{m}{M}$$

and

$$\hat{P} \equiv \frac{P}{M'} = \frac{P}{gM}.$$

From equation (5.3), we then get

$$c_1 = \frac{\hat{m}M + (g-1)M}{\hat{P}gM} = \frac{\hat{m} + g - 1}{g\hat{P}}. \tag{5.5}$$

We can then substitute for c_1 in equation (5.4) and get

$$\begin{aligned} c_2 &= wh + rk - x + \frac{\hat{m}M}{\hat{P}gM} + \frac{(g-1)M}{P} - \frac{\hat{m} + g - 1}{g\hat{P}} - \frac{\hat{m}'gM}{\hat{P}gM} \\ &= wh + rk - x - \frac{\hat{m}'}{\hat{P}}. \end{aligned} \tag{5.6}$$

From now on, I will disregard of the "hats" and use m and P instead of $\hat{m}$ and $\hat{P}$ respectively.

Now, the agents' problem can be specified as the dynamic optimization problem

$$v(\tilde{z}, K, k, m) = \max_{\{d, m'\}} \{u(c_1, c_2, h) + \beta \mathrm{E} v(\tilde{z}', K', k', m')\} \tag{5.7}$$

subject to

$$u(c_1, c_2, h) = \alpha \ln c_1 + (1-\alpha) \ln c_2 - \gamma h,$$

(5.5) and (5.6), where, if we temporarily ignore the money growth process, the exogenous state variables are $\tilde{z} = \begin{bmatrix} 1 & z \end{bmatrix}'$, the endogenous state variable is k, and the decision variables are $d = \begin{bmatrix} x & h \end{bmatrix}'$. The dynamics of this economy are given by

$$\tilde{z}' = \begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix} \tilde{z},$$

and

$$k' = B(k, x) \equiv (1-\delta) k + x.$$

I solve the deterministic version of this problem and then make a quadratic-linear approximation around the steady state to solve the stochastic problem.

2.2 Nominal rigidities

In order to get interesting effects of monetary policy, I introduce a nominal rigidity. More specifically, I assume that nominal wages are set before contemporaneous productivity shocks are observed. The nominal wage, W^c, is set equal to the expected marginal product of labor (in nominal terms), i.e.

$$W^c = E\left\{P(1-\theta) e^z \left(\frac{K}{H^e}\right)^{\theta}\right\},$$

where H^e is the expected labor demand. After the shocks have been revealed firms decide on labor demand. Now W^c is given, so the labor input chosen by firms is that which makes the marginal product of labor equal to the real wage, i.e. firms choose H such that

$$\frac{W^c}{P} = (1-\theta) e^z \left(\frac{K}{H}\right)^{\theta}.$$

Combining these two equations, taking logs, approximating the logs of expected values with expected values of logs, and solving for H results in

$$\ln H = \mathrm{E} \ln H + \frac{1}{\theta} (\ln P - \mathrm{E} \ln P) + \frac{1}{\theta} (z - \mathrm{E} z). \tag{5.8}$$

As earlier, we work with transformed prices, $\hat{P} = \frac{P}{M'}$. To be able to reformulate (5.8) in terms of transformed prices we note that, $\ln P = \ln \hat{P} + \ln M'$, and $\ln M' = \ln M + \mu$. Now,

$$\ln M' - \mathrm{E} \ln M' = \mu - \mathrm{E}\mu.$$

We also know that $z - Ez = \varepsilon$. From this we get

$$\ln H = \mathrm{E} \ln H + \frac{1}{\theta} \left(\ln \hat{P} - \mathrm{E} \ln \hat{P} \right) + \frac{1}{\theta} \varepsilon + \frac{1}{\theta} \left(\mu - \mathrm{E}\mu \right). \tag{5.9}$$

2.3 Central bank behavior and money supply

I assume that the central bank can commit to follow a rule which is linear in the state variables, and which is decided before any realizations of the state variables are observed. When deciding on the rule, the central bank takes into account the effects its decision has on the behavior of the agents in the economy. The bank also uses its knowledge of the probability distribution for the future state variables and how these depend on the policy rule it chooses. Given the linear approximation of the agents' decision rules, and the approximation $\ln(1+\pi) \approx \pi$, the quadratic loss function implies that the optimal money supply rule under commitment would be linear in the state variables if the central bank did not take into account the effects its actions have on agents' decision rules. Here, however, the central bank does consider changes in agents' decision rules when deciding on the policy rule. Therefore, the optimal linear rule need not be the same as the rule chosen when the central bank can commit to follow *any* rule.

A natural question to ask at this point is if this kind of rule realistically captures the actual behavior of central banks. I will not claim that it does. My rationale for using it is that it is simpler to model. I also believe that the linear rule is a good approximation of the optimal rule with no linearity restriction. Moreover, as suggested e.g. in Currie and Levine (1993), it is difficult for the public to monitor the central bank's fulfillment of a complex rule. If that is the case, committing to follow the complex rule might be impossible. When information is incomplete and learning is important, a simple rule might outperform a more complex rule.

As mentioned earlier, the central bank is assumed to have preferences for inflation and output stability. It conducts monetary policy to minimize a weighted sum of the unconditional variances of inflation and output,

$$L = \frac{1}{2}\mathrm{E}\left[\left(\pi_t - \pi^*\right)^2 + \lambda\left(y_t - y^*\right)^2\right], \tag{5.10}$$

where π^* is the central bank's inflation target, y_t is log output, and y^* its output target. The output target is assumed to be the logarithm of output in the steady state where $\pi = \pi^*$. Inflation is given by

$$\pi_t = \frac{P_t}{P_{t-1}} - 1 = \frac{\hat{P}_t M_{t+1}}{\hat{P}_{t-1} M_t} - 1 = e^{\mu_t}\frac{\hat{P}_t}{\hat{P}_{t-1}} - 1.$$

The central bank's loss function is not motivated by maximization of agents' utility. In particular, nothing in the model can rationalize positive nominal interest rates or inflation. Also, the desire to stabilize output around the steady state might look like a strange objective. However, agents prefer a smooth level of consumption, and for a given average level of inflation, all the central bank will do to stabilize output is to shift inflation over time. Due to the wage rigidity, hours worked will overreact to productivity shocks. The central bank will mitigate these overreactions and it will shift the distortions from the inflation tax to times when consumption is high. This will not influence the average level of output or consumption. Simulations show that the agents' average utility is slightly increasing in λ, at least for $\lambda \in [0, 0.5]$.

The central bank sets the money growth rate, μ_t^{CB}, after observing the productivity shock, ε_t. We also assume that the bank does not have perfect control over the money growth rate, so realized money growth is given by

$$\mu_t = \mu_t^{CB} + \xi_t,$$

where ξ is Gaussian white noise with variance σ_ξ^2.[3] This setup results in the

[3] Cooley and Hansen assume that ξ is log-normally distributed in order to ensure that

following decision rule for the money growth rate[4],

$$\mu_t^{CB} = \beta_1 + \beta_2 z_{t-1} + \beta_3 \varepsilon_t + \beta_4 \ln K_t + \beta_5 \ln \hat{P}_{t-1}. \tag{5.11}$$

I thus assume that the central bank sets money supply at time t after having observed the contemporaneous productivity shock, ε_t, but not the money growth shock, ξ_t. Agents on the other hand observe both shocks and thereby also μ_t before they have to make their decisions for hours worked and consumption.

2.4 Equilibrium and solution

I solve the model by making a linear quadratic approximation around the steady state. The equilibrium then consists of a matrix $\boldsymbol{\alpha}$ describing the dynamics of capital, labor supply, and prices, and a decision rule for money supply, $\boldsymbol{\beta}$. More specifically the dynamics of the economy are determined by the following two equations in addition to the exogenous process for productivity,

$$\begin{bmatrix} \ln K_{t+1} \\ \ln H_t \\ \ln P_t \end{bmatrix} = \boldsymbol{\alpha} \begin{bmatrix} 1 \\ z_{t-1} \\ \varepsilon_t \\ \xi_t \\ \ln K_t \\ \ln \hat{P}_{t-1} \end{bmatrix}$$

money growth is always positive. They thereby guarantee that the cash-in-advance restriction binds. There is no point for me to make the same assumption since the derived money supply rule (5.11) will allow money growth to be negative anyway. I assume that the cash-in-advance restriction binds and disregard the problem.

[4] With exogenous money supply, the state variables for agents would be z_{t-1}, ε_t, $\ln K_t$, ξ_t, and possibly previous realizations of money growth. Here, the central bank does not observe ξ_t when deciding on μ_t^{CB}. Moreover, $\ln \hat{P}_{t-1}$ is needed as a state variable to calculate inflation in the loss function.

$$\mu_t = \boldsymbol{\beta} \begin{bmatrix} 1 \\ z_{t-1} \\ \varepsilon_t \\ \ln K_t \\ \ln \hat{P}_{t-1} \end{bmatrix} + \xi_t.$$

The equilibrium conditions are:

– For a given $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$ is consistent with optimization of firms and individuals. Hence, $\boldsymbol{\alpha}$ can be thought of as a function of $\boldsymbol{\beta}$.

– The money supply rule $\boldsymbol{\beta}$ solves the central bank's optimization problem (5.10), $\min_{\boldsymbol{\beta}} L(\boldsymbol{\alpha}(\boldsymbol{\beta}), \boldsymbol{\beta})$.

To solve this problem in practice, I rely on numerical methods. The algorithm is as follows. First, guess some $\boldsymbol{\beta}$ and solve for $\boldsymbol{\alpha}$.Then evaluate the central bank's loss function for this money supply rule. Next, (numerically) differentiate the loss function with respect to the elements in the vector $\boldsymbol{\beta}$. Finally, use some numerical minimization method to update the candidate solution to the minimization problem. The problem appears to be very non-linear, and at least for high values of λ, the solution is sensitive to the initial guess of β. I have thus experimented with a variety of initial values.

To solve the representative agent's problem for a given money supply rule I do as follows.[5] First, solve the problem without wage rigidities. This is a standard real business cycle exercise. Let $\tilde{\boldsymbol{\alpha}}$ denote the decision rules then obtained, i.e.

$$\begin{bmatrix} \ln \tilde{K}_{t+1} \\ \ln \tilde{H}_t \\ \ln \tilde{P}_t \end{bmatrix} = \tilde{\boldsymbol{\alpha}} \begin{bmatrix} 1 \\ z_{t-1} \\ \varepsilon_t \\ \xi_t \\ \ln \tilde{K}_t \\ \ln \tilde{P}_{t-1} \end{bmatrix}.$$

[5] This exactly follows Cooley and Hansen (1995).

Next, note that the linear quadratic approximation used in obtaining the above solution imposes certainty equivalence on the problem. Therefore $\mathrm{E}\ln H = \mathrm{E}\ln \tilde{H}$, and $\mathrm{E}\ln \hat{P} = \mathrm{E}\ln \tilde{P}$. By using these equalities in (5.9) we can solve for hours actually worked to get the actual $\boldsymbol{\alpha}$. The money supply rule is implicit in the solution to the agent's problem. The state variable $\ln \hat{P}_{t-1}$ together with the other state variables provide sufficient information for agents to make the best prediction of money growth at t. To solve this economy I also use μ as a state variable, even though it will not appear in the decision rules. It is needed to calculate hours actually worked from equation (5.9).

3 Business cycle properties of the model

In this section, the general business cycle behavior of the model is discussed and evaluated. The main conclusion is that the model captures many, but not all, features of the business cycle, and that it behaves at least as good as other models in the field.

As far as possible, the model is calibrated with values from Cooley and Hansen (1995). In addition to those values, we must specify the standard deviation of the money supply shocks, σ_ξ, and the weight on output stability in the central bank's loss function, λ. It turns out that letting $\sigma_\xi = 0.0089$, the same value as in Cooley and Hansen, yields a standard deviation of μ which is close to 0.0089 even though Cooley and Hansen's money growth process is totally different from the one used in this model. Rather arbitrarily, I first let $\lambda = 0.1$ and $\lambda = 0.5$. Those values make the central bank's average loss from output fluctuations approximately of the same size as the average loss from inflation fluctuations. The parameter values used are summarized below.

Parameter Values

$$\alpha = .84 \quad \beta = .989 \quad \gamma = 2.53 \quad \delta = .019 \quad \theta = .40 \quad \rho = .95 \quad \sigma_\varepsilon = .007$$

The business cycle properties of the real variables in the U.S. economy are well known. All of them are highly procyclical, except the capital stock which is acyclical. Investment is much more volatile than output and hours worked, which in turn are more volatile than consumption and the capital stock. Productivity leads the business cycle slightly while all other real variables peek at the same time as production. The most important feature of the nominal variables is that prices are countercyclical while inflation and money are procyclical. Inflation lags the business cycle but prices and money lead the cycle. Tables 1 and 2 summarize the business cycle statistics for the U.S. economy and for simulated economies with $\lambda = 0.1$ and $\lambda = 0.5$. All statistics reported, both for the U.S. economy and for the model economies, are calculated on detrended variables.[6]

Responses to productivity shocks in the simulated economies are reported in Figures 1 and 2. The solid lines in these graphs show impulse-responses for an economy where money supply is exogenously specified to be autocorrelated and stochastic (Cooley and Hansen's model), while the dashed lines show the impulse-responses for economies with $\lambda = 0.1$ and $\lambda = 0.5$. Tables 1 to 3 report volatilities and correlations of the variables in these economies. We see that the model generates procyclical inflation. Prices are countercyclical when λ is high but almost acyclical when λ is low. The nominal interest rate is acyclical. The correlation between consumption and output is close to that in the U.S. economy and consumption and output are not as volatile as in Cooley and Hansen's model. In contrast, Huh's (1993) model generates a counterfactually high correlation between output and consumption (0.98). Also, the behavior of the nominal interest rate and the leads and lags of nominal variables are more satisfactory than in Huh's model.

Nelson (1997) points out that most equilibrium models with money fail to capture two properties of the U.S. economy, namely that inflation reacts to money shocks with a lag and that inflation is persistent. The statistics in Table 3 show

[6] The variables were detrended with an H-P filter with $\lambda = 1600$.

that this critique certainly applies to the current model when λ is low. When λ is high, nominal variables become more serially correlated since the central bank then uses intertemporal changes in the inflation tax to stimulate the economy in recessions and to depress it in booms. However, inflation still reacts to money growth shock without delay.

I see several reasons for why we should expect the serial correlations of nominal variables to be lower in the model than in the U.S. economy. First, the positive autocorrelation for money in the U.S. is calculated from quarterly data on M1 growth from 1959:II to 1996:II. It is not probable that the federal reserve has had a constant inflation target during that period. If we introduce an inflation target that shifts over time, money growth and inflation will become more autocorrelated provided that the inflation target is serially correlated. The existence of structural changes in the Federal Reserve's behavior is supported empirically by e.g. Bernanke and Mihov (1995), Gavin and Kydland (1996), and Clarida, Galí, and Gertler (1997).

Another explanation could be that the central bank's loss function is fundamentally different from the one used in this paper. If, for example, there are costs for the society associated with changes in the inflation rate in addition to changes in the price level, the central bank will not restore inflation to the normal level immediately after it has reacted to a shock. This explanation is a bit hard to motivate in an environment where agents are rational.

Finally, the nominal rigidity only lasts for one period in the model. There is therefore no reason for the central bank to pursue an expansionary policy in response to a negative productivity shock even if the shock has long-lasting effects on real variables. It would be more realistic to allow wage contracts to last for more than one quarter. By doing that, we would also derive money supply rules with more serial correlation.

4 Effects of changes in central bank preferences

In this section, I examine the effects that changes in the central bank's preferences have on its own policy rule, on the agents' decision rules, and on the properties of the resulting economies. I first look at changes in the weight the central bank puts on inflation stability relative to output stability. The central bank's inflation target, π^*, is assumed to be 1.5% per quarter.

Table 1 shows that the central bank does have *some* control over the two components in its loss function – the volatility of output and inflation. As the weight put on output stability increases from 0.1 to 0.5, the coefficient of variation for output falls from 2.09 to 1.87 and the coefficient of variation for inflation increases from 0.88 to 1.09. In general, however, the volatility of variables does not change much when λ changes. The correlations between output and some nominal variables reported in Table 2 change considerably when λ change. This holds in particular for the correlation between output and prices, but also for the leads and lags of money and inflation.

In Tables 4 and 5, I report optimal decision rules for different values of the weight λ. With the numerical methods used to solve for the decision rules, some of the parameters are difficult to solve for with good precision. This holds in particular for β_1 and β_4 (the constant term and capital).[7]

The most interesting finding of these tables is the central bank's response to productivity shocks. When the central bank puts much weight on inflation stability, it increases money supply in response to positive productivity shocks. This is because these shocks tend to drive inflation down. But positive productivity shocks also tend to increase output. Therefore, the central bank will contract money supply and exploit the short run Phillips curve when it puts more weight on output stability. In Figures 1 and 2, we see the results that these different money supply rules have on the agents' behavior. When λ is low, hours worked

[7] The source of this problem is probably that the capital stock does not fluctuate much. Hence, it is difficult to separate it from the constant term.

increases in response to positive productivity shocks, but when λ is high, the initial response of hours worked to these shocks is small. With the exception of hours worked, the impulse-response graphs also confirm that the money supply process is not important for the behavior of real variables, but that nominal variables behave differently under different monetary regimes.

Money supply shocks have roughly the same effects under all monetary regimes. Since wages are set before money growth shocks are observed, output, hours worked, and investment increase significantly in response to positive shocks, and so do inflation and nominal interest rates. As can be seen in Figures 3 and 4, though, the effects are very transitory. In the period after the shock, most variables are back to their trend levels, so money supply shocks cannot account for the cyclical behavior we observe in real variables.

So far, I have assumed that the public has perfect information about the central bank's preferences and that agents immediately understand what policy rule the central bank will use. In reality, central bank preferences might change over time and it is possible that these changes are not immediately noticed or understood by the public. Moreover, even if the new preferences are taken into account immediately, agents might be afraid that the central bank's preferences will change again. The Volcker era is arguably a period with considerable uncertainty about monetary policy, for instance whether the shift to low inflation was persistent or not. If we allow for mechanisms like these, monetary policy can cause cyclicality in real variables since agents' misapprehensions or mistrust will be serially correlated in itself.

The model I use here provides a tool for thinking about central bank preferences in a business cycle framework, but the complexity of the model does not allow us to explicitly introduce a new dimension of uncertainty. To get an upper limit of the quantitative effects that this uncertainty can induce, I have looked at changes in the central bank's preferences which are not noticed by the agents. I find that if $\lambda = 0.1$ and the central bank's inflation target falls from 1.5 to 0.75

percent per quarter, and if this change is not perceived by the agents, output falls immediately to approximately 1 percent below trend, hours to 1.5 percent below trend, and investment to 4 percent below trend. Consumption does not react much initially.[8]

5 Concluding remarks

The effects of anticipated and unanticipated monetary policy have for a long time been a controversial issue in economics. The observation that correlations between real and nominal variables are significant in magnitude is not enough for us to conclude a causality from money to output or vice versa. Theoretically, these correlations can, for example, be due to nominal rigidities, i.e. that money causes output. It could also be the case that money demand responds to real activity, i.e. that real variables cause fluctuations in nominal variables. In models trying to explain the money - output correlations, money supply has often been neglected. In this paper, I have worked from the starting point that the money supply process is the most important source of fluctuations in nominal variables. Therefore, money supply will also be an important factor behind the relationship between real and nominal variables if, which seems to be the case, the central bank takes the real economy into account when deciding on money supply.

In order to study these issues, this paper has endogenized the central bank's money supply decisions in a dynamic general equilibrium model of macroeconomic fluctuations. The central bank has some power to stabilize inflation and output in the model. To achieve this stabilization it has to react to changes in the real and nominal environment. I find that the money supply process, as expected, is an important determinant of the joint behavior of real and nominal variables. I also find that small changes in the central bank's desire to stabilize output

[8] It is worth noting that if the central bank's preference change were noticed by the public, all these variables would increase by approximately 1 percent since the distorting effect of inflation would decrease.

relative to inflation cause large changes in the implied money supply rule and in the behavior of nominal variables, but real variables are mostly unaffected.

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Table 1: *Volatility*

Variable	U.S.	$\lambda = 0.1$	$\lambda = 0.5$
Output	1.72	2.09	1.87
Consumption	0.86	0.43	0.43
Investment	8.24	7.50	6.58
Capital stock	0.63	0.39	0.34
Hours	1.59	2.44	2.27
Productivity	0.90	0.93	1.04
Prices[1]	1.43	1.10	1.62
Inflation	0.57	0.88	1.09
Nominal interest rate[2]	1.29	0.50	0.43
Real interest rate	*n.a.*	0.04	0.04
Money[3]	0.84	0.85	0.93
Velocity[4]	1.94	1.79	1.57
Money growth	.009	.009	.012

Notes: U.S. data adapted from Cooley and Prescott (1995), Cooley and Hansen (1995) and Hansen (1985). Volatility is measured as the standard deviation of percentual fluctuations around trend.
[1] CPI for U.S. data, [2] TB1MO for U.S. data,
[3] Monetary base for U.S. data, [4] Velocity of M1 for U.S. data.

Table 2: *Correlations with Output*

	Cross-Correlation of Output(t) with:								
	$x(t-2)$			$x(t)$			$x(t+2)$		
Variable x	U.S.	$\lambda_{.1}$	$\lambda_{.5}$	U.S.	$\lambda_{.1}$	$\lambda_{.5}$	U.S.	$\lambda_{.1}$	$\lambda_{.5}$
Output	.63	.26	.28	1.00	1.00	1.00	.63	.26	.28
Consumption	.68	.14	.31	.77	.75	.76	.47	.50	.44
Investment	.59	.26	.26	.91	.99	.99	.50	.20	.23
Capital stock	*n.a.*	−.31	−.34	.04	−.05	−.08	*n.a.*	.46	.45
Hours	.53	.18	.12	.86	.93	.89	.69	.10	.15
Productivity	.30	.10	.25	.41	−.19	−.16	.00	.31	.18
Prices[1]	−.72	−.28	−.50	−.52	−.06	−.39	−.17	−.02	−.18
Inflation	.01	−.18	−.29	.34	.31	.31	.44	.08	.22
Nom. int. rate[2]	−.03	.01	−.05	.40	.04	.05	.44	.04	.12
Real int. rate	*n.a.*	.34	.21	*n.a.*	.57	−.19	*n.a.*	.37	.09
Money[3]	.42	−.04	−.18	.30	.61	.55	.15	−.04	.10
Velocity[4]	−.08	.27	.25	.37	.99	.99	.33	.18	.22

Notes: U.S. data adapted from Cooley and Prescott (1995), Cooley and Hansen (1995) and Hansen (1985). $\lambda_{.1}$ = model with $\lambda = 0.1$, $\lambda_{.5}$ = model with $\lambda = 0.5$.
[1] CPI for U.S. data, [2] TB1MO for U.S. data, [3] Monetary base for U.S. data,
[4] Velocity of M1 for U.S. data.

Table 3: *Auto-Correlations and Cross-Correlations of Nominal Variables*

	$k=0$			$k=1$			$k=2$			$k=4$		
	U.S.	$\lambda_{.1}$	$\lambda_{.5}$	U.S.	$\lambda_{.1}$	$\lambda_{.5}$	U.S.	$\lambda_{.1}$	$\lambda_{.5}$	U.S.	$\lambda_{.1}$	$\lambda_{.5}$
$\rho(\mu_t, \mu_{t-k})$	1.00	1.00	1.00	.54	.04	.37	.50	−.06	.34	.27	.14	.45
$\rho(\pi_t, \pi_{t-k})$	1.00	1.00	1.00	.84	.13	.50	.78	.00	.39	.76	.20	.45
$\rho(\pi_t, \mu_{t-k})$	.34	.86	.95	.36	−.02	.47	.38	−.07	.37	.32	.22	.52

Notes: U.S. data adapted from Nelson (1997) and own calculations. $\lambda_{.1}$ = model with $\lambda = 0.1$, $\lambda_{.5}$ = model with $\lambda = 0.5$. ρ denotes correlations, μ_t is money growth, and π_t is inflation.

Table 4: *Central Bank Decision Rules and Resulting Economies*
$\mu_t^{CB} = \beta_1 + \beta_2 z_{t-1} + \beta_3 \varepsilon_t + \beta_4 \ln K_t + \beta_5 \ln \hat{P}_{t-1}$

						Properties of the economy		
λ	const.	z_-	ε	$\ln K$	$\ln \hat{P}_-$	SD%(Y)	SD%(π)	Corr(μ)
0.0	0.338	−0.001	0.422	−0.107	−0.140	2.25	0.86	0.00
0.1	0.204	−0.035	0.099	−0.065	−0.189	2.09	0.88	0.04
0.2	0.112	−0.065	−0.130	−0.038	−0.259	1.99	0.94	0.12
0.5	−0.391	−0.105	−0.501	0.120	−0.342	1.87	1.09	0.37
1.0	−1.342	−0.038	−0.738	0.425	−0.258	1.80	1.25	0.55

Notes: SD% is the standard deviation of a variable's percentual fluctuations relative to its trend. Y is output and π is inflation. Corr(μ) is the autocorrelation of money growth.

Table 5: *Decision Rules for Agents*
$x_{i,t} = \alpha_{i,1} + \alpha_{i,2} z_{t-1} + \alpha_{i,3} \varepsilon_t + \alpha_{i,4} \xi_t + \alpha_{i,5} \ln K_t + \alpha_{i,6} \ln \hat{P}_{t-1}$

x	λ	const.	z_-	ε	ξ	$\ln K$	$\ln \hat{P}_-$
	0.0	2.05	−0.38	−0.43	−0.06	−0.68	−0.137
	0.1	1.97	−0.43	−0.43	−0.06	−0.66	−0.185
$\ln \hat{P}$	0.2	1.93	−0.48	−0.43	−0.07	−0.65	−0.253
	0.5	1.57	−0.56	−0.45	−0.07	−0.54	−0.335
	1.0	0.81	−0.55	−0.49	−0.08	−0.29	−0.251
	0.0	0.15	0.11	0.16	0.10	0.95	−0.006
	0.1	0.15	0.11	0.13	0.10	0.95	−0.008
$\ln K'$	0.2	0.15	0.11	0.10	0.10	0.95	−0.011
	0.5	0.15	0.10	0.06	0.10	0.95	−0.015
	1.0	0.14	0.09	0.02	0.10	0.95	−0.011
	0.0	0.23	1.42	2.48	2.34	−0.46	0.008
	0.1	0.36	1.38	1.68	2.34	−0.49	0.011
$\ln H$	0.2	0.48	1.34	1.11	2.34	−0.53	0.014
	0.5	0.83	1.24	0.13	2.33	−0.64	0.019
	1.0	1.34	1.08	−0.57	2.31	−0.80	0.016

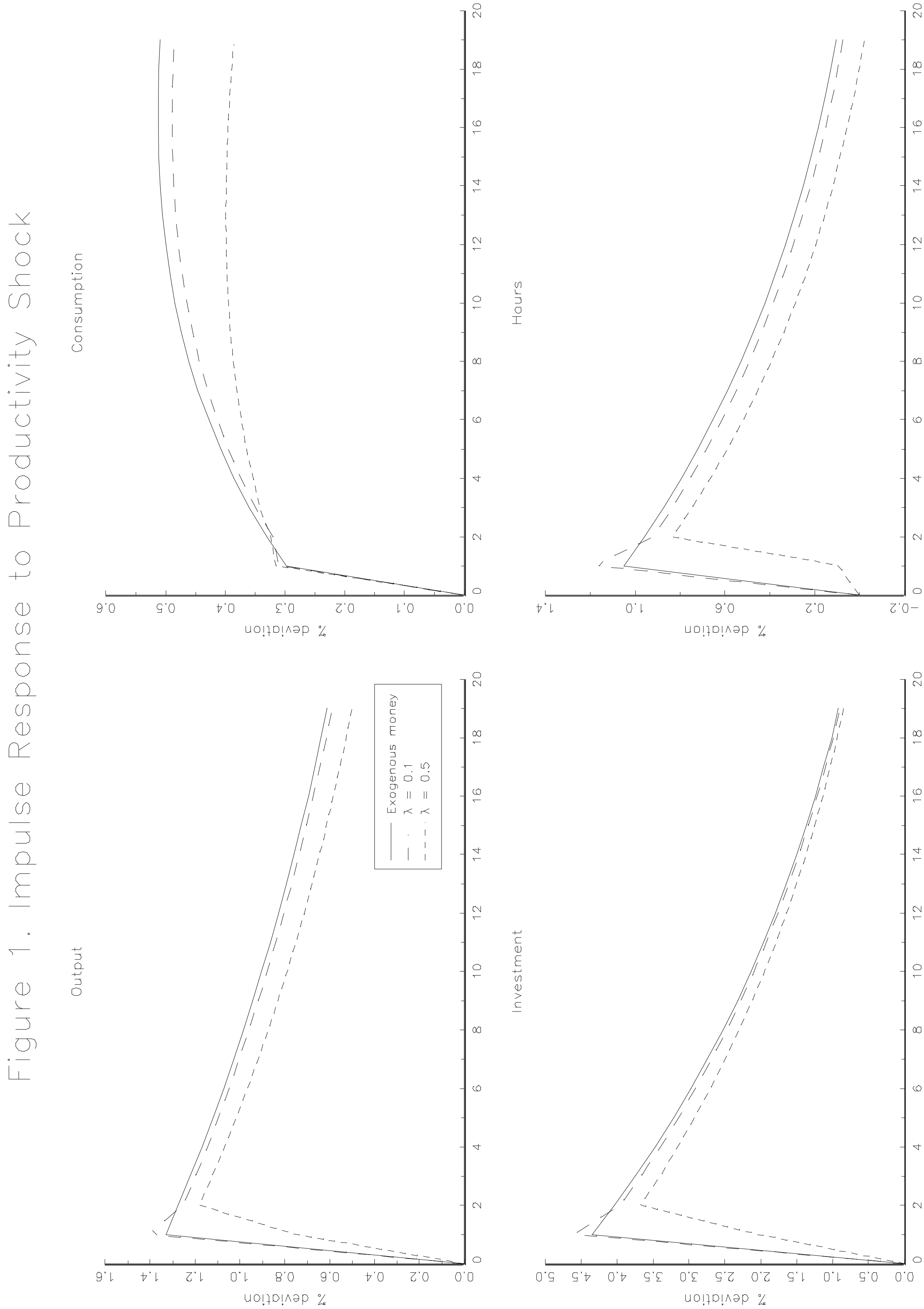
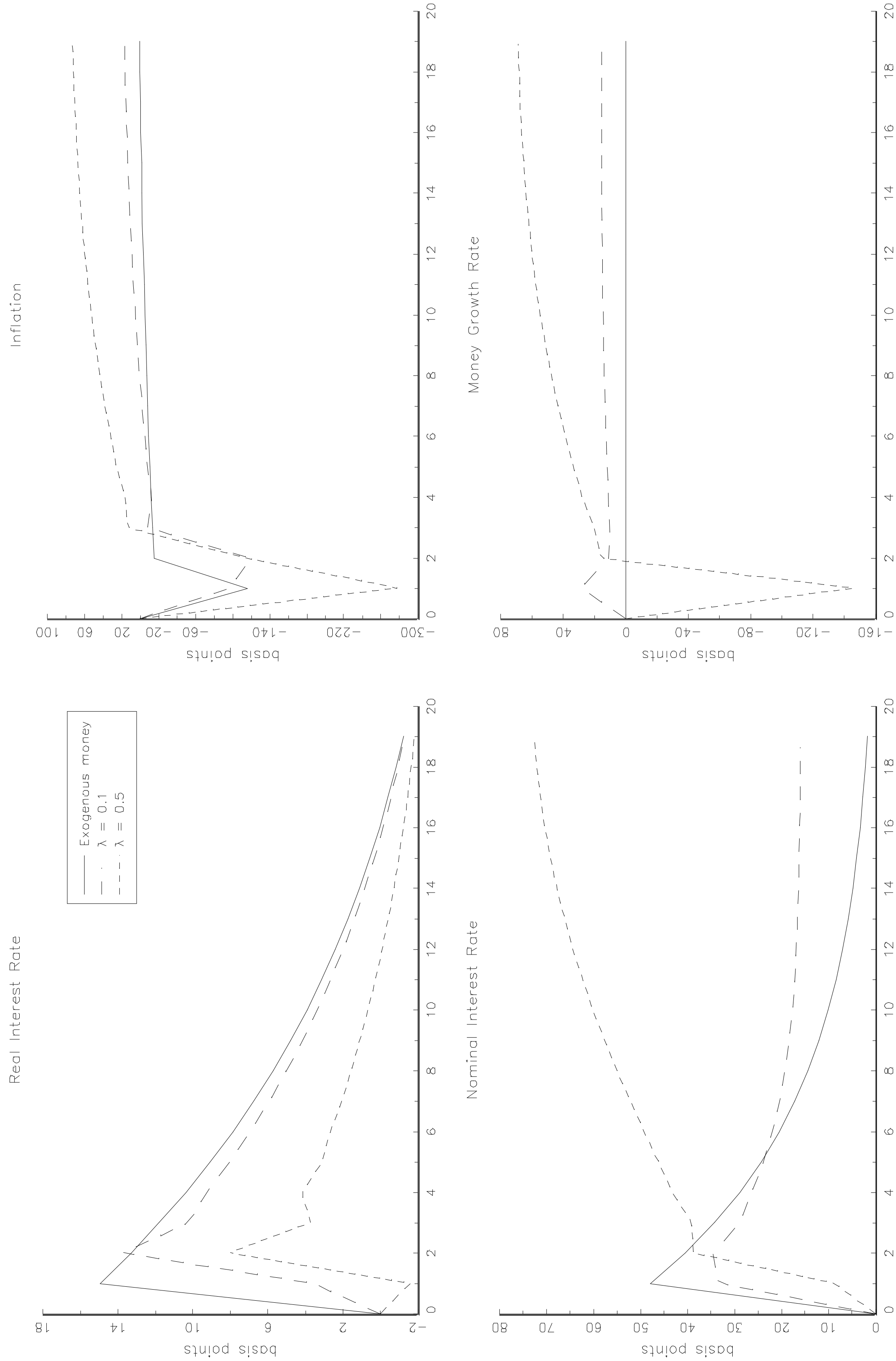


Figure 1. Impulse Response to Productivity Shock

Figure 2. Impulse Response to Productivity Shock



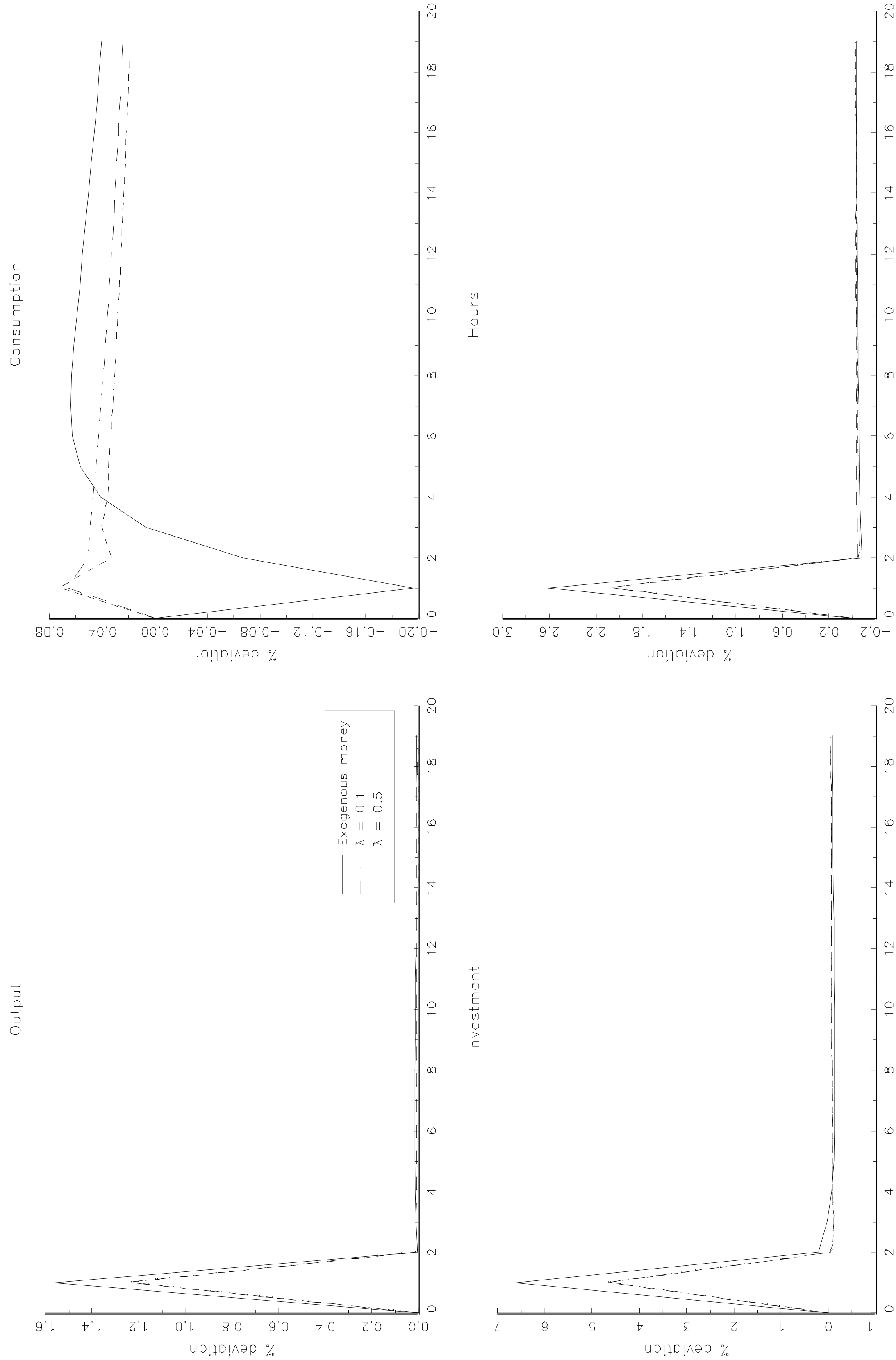


Figure 3. Impulse Response to Money Supply Shock

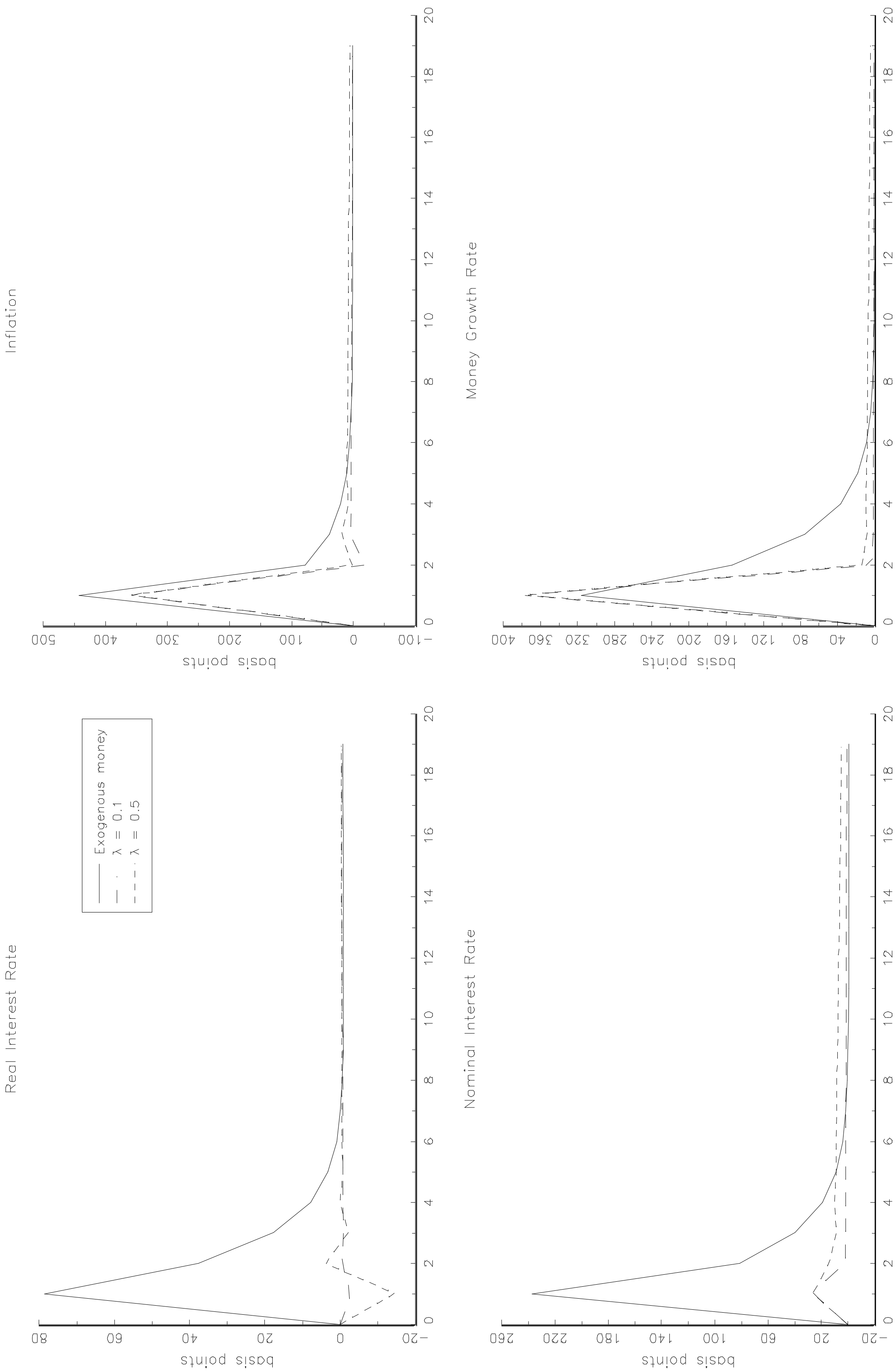


Figure 4. Impulse Response to Money Supply Shock