The Time Consistency Problem of Monetary Policy under Alternative Supply Side Modeling

Martin Flodén*
Stockholm School of Economics†

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Abstract

This paper examines how sensitive the results in the literature on time consistent monetary policy are to changes in the supply side modeling. It is found that a variety of alternative models of the supply side yield either the same or very similar results as the traditional model does. There is always an inflation bias under discretion and a stabilization bias under discretion in the presence of output persistence. Moreover, the imposition of a state-contingent linear inflation contract makes the central bank act as if it could commit to an optimal policy rule for inflation.

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†Department of Economics, Box 6501, S-113 83 Stockholm, Sweden. E-mail: nemf@hhs.se, phone: + 46 8 736 96 48, fax: + 46 8 31 32 07.

1. Introduction

Inspired by Kydland & Prescott (1977), Barro & Gordon (1983) and Rogoff (1985), a considerable literature on central bank behavior and its time consistency problem has been developed during the last decade. Common to most of these articles is that they have employed a quadratic loss function and assumed that monetary policy has effects via a traditional Phillips curve.[1] The objective of this paper is to examine the sensitivity of the obtained results to the specific formulation of the Phillips curve. I will use different kinds of supply side modelings to end up with two variants of the Phillips curve. These Phillips curves will be used in a framework similar to that of Svensson (1995).

In their seminal paper, Kydland and Prescott noted that there will be an inflation bias when the central bank acts under discretion, i.e. the equilibrium inflation will be higher than the optimal inflation rate when the central bank does not internalize the effects its behavior has on expectations. The intuition behind this result is that surprise inflation will raise output in the short run. If inflation is expected to be on its target level, the marginal cost of inflation is lower than the marginal benefit inflation surprises render in terms of increased output. Equilibrium will be obtained at the level of inflation expectations where the marginal benefit of increased output equals marginal cost of increased inflation. This requires inflation to be higher than the target rate. However, since the agents are assumed to have rational expectations, output will only respond to contemporaneous shocks and it will on average be at its natural level. A better outcome for society would be obtained if the central bank could commit to set the inflation rate according to a specific rule. In that case, output would follow the same path as under discretion, but inflation would on average be at its target level.[2]

Some recent papers, e.g. Lockwood & Philippopoulos (1994), Lockwood, Miller & Zhang (1995) and Svensson (1995) allow for persistence in output (or in unemployment). A general result from these papers is that output will not follow the same path under discretion as under commitment. Instead, inflation will be too volatile and output too stable under discretion. The reason is that low output is not only bad per se, but that it also increases the inflation bias under discretion. Therefore it becomes more important to avoid output fluctuations under discretion than under commitment. Svensson (1995) refers to this property as a stabilization bias.

Lately, Persson & Tabellini (1993), Walsh (1995) and Lockwood, Miller & Zhang (1995) have shown how to formulate central bank contracts which enable optimal monetary policy to be combined with time consistency. Previous papers, e.g. Rogoff (1985) and Flood & Isaard (1989) have suggested improvements from the discretionary scenario, but these improvements did not result in the social optimum. The papers by Persson and Tabellini and by Walsh show that a simple linear inflation contract implicitly will make the central bank internalize its effects on inflation expectations so that it will act as under commitment. Lockwood, Miller and Zhang have further developped this idea and allowed for persistence in output. They show that in the presence of output persistence, the inflation contract must

[1] Persson & Tabellini (1993) is an exception. In that paper, the loss function is of a general form.

[2] Kydland and Prescott illustrated their result with a deterministic example, but later papers typically allow for supply shocks.

be contingent on previous output to render optimality – when output is low, inflation shall be less penalized. Svensson (1995) shows that the same results can be obtained if a state-contingent inflation target is implemented instead of a state-contingent inflation contract.

The three results reported above (the inflation bias under discretion, the stabilization bias under discretion in the presence of output persistence and the possibility to obtain optimality with state-contingent linear inflation contracts) are those I find fundamental and important in the literature on the time consistency problem of monetary policy. This paper will be devoted to examining the robustness of these results to changes in the specification of the supply side modeling.

In the next section, I will describe the basic model setup. I then briefly summarize the results from the model with a traditional Phillips curve. In section 4, I explore the effects of using New Keynesian assumptions about wage and price formation. The following section extends one of the New Keynesian models in section 4 by allowing for long term nominal contracts. In section 6, I introduce persistence to the analysis. Finally, section 7 concludes.

2. Model

The society and the central bank are assumed to have preferences over the inflation rate and the output level. The loss function at time t is

$$L_t = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2\right], \qquad (1)$$

where π_t is the inflation rate at time t, π^* is the desired inflation rate, y_t is the deviation of output from its trend level and y^* is the desired deviation of output from its trend level. The social loss function over time is then given by

$$V = E_0 \sum_{t=1}^{\infty} \beta^{t-1} L_t, \qquad (2)$$

where β is the discount factor.

Throughout the analysis, the central bank is assumed to have the same preferences as society. It is also assumed to have perfect control over the inflation rate. The inflation rate at time t is set after the central bank has observed the supply shock, ϵ_t. When the supply shock has been observed, but before the central bank has set the inflation rate, π_t, private agents form expectation for future periods, i.e. of π_{t+1}, π_{t+2}, etc. This timing will yield the same results as the more realistic modeling where agents form expectations simultaneously with the central bank's action.[3]

[3] The setup I have in mind is: First ϵ_t is observed by all agents. Second, private agents and the central bank form expectations and acts respectively. Third, still in the beginning of period t, if either the agents or the central bank is dissatisfied with its decision, we repeat at the second step. This continues until both sides are content with their decisions.

3. Traditional Phillips curve

In order for monetary policy to have effects, there must be some rigidity or imperfection on the market. In the time consistency literature, the imperfection has usually been modeled as wage and price rigidities yielding a traditional expectations augmented Phillips curve in the spirit of Friedman and Phelps. Firms set wages and prices at time t before observing the productivity shock, ϵ_t, whilst the central bank sets inflation at t after having observed the shock. Wages and prices are therefore based on inflation expectations, which results in the Phillips curve

$$y_t = \rho y_{t-1} + \alpha \left(\pi_t - \pi^e_{t,t-1} \right) + \epsilon_t, \tag{3}$$

where ρ is a measure of the degree of persistence in output and $\pi^e_{t,t-1} \equiv E_{t-1}\pi_t$ is the expectation of inflation in period t, formed in period $t-1$. The implications and results of this setup have been comprehensively analyzed and surveyed in Svensson (1995). I will briefly summarize these results to use for later reference.

With this setup of the model, there is obviously a time consistency problem. Given inflation expectations, surprisingly high inflation will increase output. Since agents are rational, they will take the time consistency problem into account when forming expectations.

We assume that the central bank wishes to minimize the social loss function (2). In the absence of output persistence, the problem is static and equivalent to the central bank's minimization of equation (1).

3.1. Commitment

Under commitment, we assume the central bank can commit to follow some policy rule for inflation. The problem is then to solve the Bellman equation

$$V(y_{t-1}) = \min_{\pi_t, \pi^e_{t,t-1}} E_{t-1} \left\{ \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right] + \beta V(y_t) \right\} \tag{4}$$

subject to (3).

The solution to this problem is the reaction function[4]

$$\pi_t = a^c - b^c \epsilon_t,$$

where

$$a^c = \pi^* \text{ and } b^c = \frac{\lambda\alpha}{1 + \lambda\alpha^2 - \beta\rho^2}.$$

This implies

$$y_t = \rho y_{t-1} + (1 - \alpha b^c)\, \epsilon_t.$$

[4] See Svensson (1995).

3.2. Discretion

Under discretion, the central bank does not internalize the effects of its behavior on expectations. The bank solves the Bellman equation

$$V(y_{t-1}) = \min_{\pi_t} \left\{ \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right] + \beta E_t V(y_t) \right\} \tag{5}$$

still subject to (3) but with inflation expectations, $\pi^e_{t,t-1}$, given.

The solution is

$$\pi_t = a^d - b^d \epsilon_t - c^d y_{t-1}, \tag{6}$$

where

$$a^d = \pi^* + \frac{\lambda \alpha y^*}{1 - \beta\rho - \beta\alpha c^d},\ b^d = \frac{\lambda\alpha + \beta\alpha \left(c^d\right)^2}{1 + \lambda\alpha^2 - \beta\rho^2 + \beta\alpha^2 \left(c^d\right)^2}$$

and

$$c^d = \frac{1 - \beta\rho^2 - \sqrt{\left(1 - \beta\rho^2\right)^2 - 4\lambda\beta\alpha^2\rho^2}}{2\beta\alpha\rho}.$$

When there is no persistence, i.e. when $\rho = 0$, the expressions are

$$a^d = \pi^* + \lambda\alpha y^*,\ b^d = \frac{\lambda\alpha}{1 + \lambda\alpha^2} \text{ and } c^d = 0. \tag{7}$$

3.3. Properties of inflation and output

From the obtained reaction functions above, we see that when the central bank can commit to follow an optimal policy rule, inflation will on average be at the desired level. Both inflation and output will fluctuate in response to supply shocks, but these fluctuations are optimal (given the labor market distortion and the output persistence). This scenario will be referred to as the second best solution.[5]

Further, we note that if the central bank cannot credibly commit, there will be a positive inflation bias. If there is no persistence in output, both inflation and output will follow the same path as under commitment, except that the path for inflation will be at a higher level under discretion than under commitment. If there is persistence, however, output will be more stable and inflation more volatile under discretion than under commitment.

Persson & Tabellini (1993) and Walsh (1995) have shown that a second best solution can be obtained under discretion if the central bank's loss function can be altered. The government (for example) should punish the central bank if inflation is too high, given previous output. By implementing a state-contingent linear inflation contract, the central bank's loss function becomes

$$L^b_t = \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right] + (k_0 + k_1 y_{t-1}) (\pi_t - \pi^*). \tag{8}$$

When the central bank has this loss function and works under discretion, we can find values on the constants k_0 and k_1 which make the outcome optimal for a society with the loss function given by equation (1).

[5] The first best solution would be to remove the labor market imperfection.

In the remainder of this paper, I will examine how robust these properties of inflation, output and optimal contracts are to the specific nature of the supply side modeling.

4. New Keynesian Phillips curve

In this section, I will explore the implications of replacing the traditional Phillips curve, (3), with one derived from New Keynesian time-dependent sticky price models. As shown by e.g. Roberts (1995), the models of Taylor (1979), Rotemberg (1982) and Calvo (1983) can be characterized by a common Phillips curve,

$$y_t = \alpha\left(\pi_t - E_t\pi_{t+1}\right) + \epsilon_{t.} \tag{9}$$

Taylor's model is one of staggered wage setting where firms set nominal wages for two periods. Half of them set wages in odd periods and the other half set prices in even periods. Rotemberg's model is rather different but the resulting Phillips curve is still the same. He assumes there is a quadratic cost to changing the nominal price at a particular time. Finally, Calvo's model is similar to Taylor's. At each time period, firms are assumed to be able to change their price only if they receive a random signal. These models are summarized in Roberts and in Blanchard & Fischer (1989).

I will abstract from output persistence in this section. Persistence will be introduced later in the paper when I analyze a more complicated model, of which this setup is a special case.[6]

Since the agents' expectations are formed simultaneously with the central bank's action, it is not immediately clear that the central bank faces a time consistency problem when the Phillips curve is given by (9). There is a time consistency problem though, and this arises because when the central bank works under discretion, whatever it does today it cannot influence expectations about what it will do tomorrow. Even if the bank announces that it will set inflation according to the socially optimal rule, and is observed to do so today, it is not credible that it will continue to do so in the future.

4.1. Commitment

Under commitment, we assume the central bank at time $t = 0$ can commit to follow some policy rule $g(\cdot)$ for inflation. Since there is no persistence in the economy, the optimal policy

[6] It is fairly straight forward but a bit cumbersome to obtain an analytical solution for the commitment scenario in the presence of output persistence. Since the problem is linear-quadratic, we know that the value function will be quadratic, $V(y_t) = \gamma_0 + \gamma_1 y_t + \gamma_2 y_t^2$, and that the optimal reaction function will be linear in the state variables, $\pi_t = \phi_0 + \phi_1 \varepsilon_t + \phi_2 y_{t-1}$. Under commitment, we soon find that $\phi_2 = 0$. It is then easy to solve the problem, and the solution is the same as in the traditional model. Under discretion, however, inflation will depend on lagged output and I have not been able to solve that problem analytically. Therefore, I only report the solutions in the absence of persistence.

Since the model used in this section is a special case of the Buiter-Miller model I use in section 5 and 6, I have been able to solve the model numerically. It turns out that the paths of both inflation and output will be exactly the same in this model as in the traditional model also when persistence is introduced, and this holds both under commitment and under discretion.

rule will be a function of contemporaneous shocks only, i.e.

$$\pi_t = g(\epsilon_t) \;\; \forall t.$$

If inflation at t reacted to earlier shocks, these reactions would be foreseen by the agents and therefore not have any impact on output.

The central bank has to choose the policy function *ex ante*, i.e. it chooses the function $g(\epsilon)$ that solves

$$\min_{g(\cdot)} V$$

subject to (9), which with our assumptions is equivalent to choosing $g(\epsilon)$ to solve

$$\min_{g(\cdot)} E_0 L_1 = \int_{-\infty}^{\infty} \frac{1}{2} \left\{ (g(\epsilon) - \pi^*)^2 + \lambda \left[\alpha (g(\epsilon) - \bar{g}) + \epsilon - y^* \right]^2 \right\} dF(\epsilon), \tag{10}$$

where

$$\bar{g} = E_0 g(\epsilon_1) = \int_{-\infty}^{\infty} g(\epsilon') \, dF(\epsilon').$$

We now see that this problem is equivalent to the problem with the traditional Phillips curve.[7] Somewhat surprisingly thus, the optimal policy rule will be the same in the two models. The reason for this result is that when the central bank chooses its optimal policy rule, it considers the effects the rule has on inflation expectations at *any* time (since inflation expectations are static). Therefore, it does not matter if inflation expectations are dated $t-1$ or t.

4.2. Discretion

Under discretion, the central bank is assumed not to be able to commit to follow some policy rule. This implies that the central bank's actions today do not alter expectations of its future behavior.[8] Therefore, and since we have assumed that there is no persistence in output, the central bank faces the same problem at each time t. The central bank's minimization of equation (2) is thus equivalent to its minimization of equation (1) at each time t. The problem is then stated as

$$\min_{\pi_t} L_t$$

subject to (9). The first order condition for this problem is

$$\pi_t - \pi^* + \lambda \alpha (y_t - y^*) = 0. \tag{11}$$

[7] To see this, just reformulate the traditional problem (4). The central bank can commit to follow some policy rule for inflation, $\pi_t = g(\epsilon_t)$. Its problem is to choose $g(\cdot)$ to solve

$$\min_{g(\cdot)} E_0 L_1 = \int_{-\infty}^{\infty} \frac{1}{2} \left[(g(\epsilon) - \pi^*)^2 + \lambda (y_t - y^*)^2 \right] dF(\varepsilon)$$

subject to $y_t = \alpha \left[\pi_t - \int g(\varepsilon') \, dF(\varepsilon') \right]$. This is evidently the same problem as (10).

[8] I.e. we define discretion to mean the absence of any influence of reputation and historical behavior on expectations.

Using equation (9) in equation (11), we get

$$\pi_t = \frac{1}{1+\lambda\alpha^2}\left(\pi^* + \lambda\alpha y^* + \lambda\alpha^2 E_t\pi_{t+1} - \lambda\alpha\epsilon_t\right). \tag{12}$$

Leading (12) once and taking expectations yields

$$\begin{aligned}
E_t\pi_{t+1} &= \frac{1}{1+\lambda\alpha^2}(\pi^* + \lambda\alpha y^*) + \frac{\lambda\alpha^2}{1+\lambda\alpha^2}E_t\pi_{t+2} \\
&= \frac{1}{1+\lambda\alpha^2}(\pi^* + \lambda\alpha y^*) + \frac{\lambda\alpha^2}{(1+\lambda\alpha^2)^2}(\pi^* + \lambda\alpha y^*) + \\
&\quad + \left(\frac{\lambda\alpha^2}{1+\lambda\alpha^2}\right)^2 E_t\pi_{t+3} \\
&= \frac{\pi^* + \lambda\alpha y^*}{1+\lambda\alpha^2}\left[1 + \frac{\lambda\alpha^2}{1+\lambda\alpha^2} + \left(\frac{\lambda\alpha^2}{1+\lambda\alpha^2}\right)^2 + \ldots\right] + \\
&\quad + \lim_{s\to\infty}\left(\frac{\lambda\alpha^2}{1+\lambda\alpha^2}\right)^{s-1} E_t\pi_{t+s}.
\end{aligned} \tag{13}$$

Since $\lambda > 0$, we see that

$$0 \leq \frac{\lambda\alpha^2}{1+\lambda\alpha^2} < 1.$$

Assuming that inflation expectations do not grow without bounds, this implies that the last term in (13) disappears and that

$$\begin{aligned}
E_t\pi_{t+1} &= \frac{\pi^* + \lambda\alpha y^*}{1+\lambda\alpha^2}\sum_{s=0}^{\infty}\left(\frac{\lambda\alpha^2}{1+\lambda\alpha^2}\right)^s = \\
&= \pi^* + \lambda\alpha y^*.
\end{aligned}$$

We use this to solve for the inflation rate at time t in equation (12):

$$\begin{aligned}
\pi_t &= \frac{\pi^* + \lambda\alpha y^*}{1+\lambda\alpha^2} + \frac{\lambda\alpha^2}{1+\lambda\alpha^2}(\pi^* + \lambda\alpha y^*) - \frac{\lambda\alpha}{1+\lambda\alpha^2}\epsilon_t \\
&= \pi^* + \lambda\alpha y^* - \frac{\lambda\alpha}{1+\lambda\alpha^2}\epsilon_t.
\end{aligned} \tag{14}$$

The central bank's reaction function is obviously exactly the same as in the case with the traditional Phillips curve. This is not surprising since the only difference between the two models is the timing of inflation expectations and these are not considered in the central bank's optimization problem.

4.3. Properties of inflation and output

We have seen that the policy rules, both under commitment and under discretion, are exactly the same when we use this forward-looking supply side modeling, as when we use the traditional Phillips curve. It is therefore evident that the properties of inflation and output will be the same in this scenario as in the traditional. Since the problems will remain equivalent also if we implement a linear inflation contract as specified in equation (8), the optimal solution will still be obtained with that contract.

5. Long term nominal contracting

The model used in this section is very similar to that of Currie & Levine (1993, chapter 6). Their model is inspired by Taylor's staggered price model and is similar to the model in Buiter & Miller (1983). Instead of assuming a fixed contract length of two years, Currie and Levine assume a proportion μ survives from one period to the next. The model is specified as

$$\begin{aligned} y_t &= \alpha(\pi_t - c_t) + \epsilon_t && (15) \\ c_t &= (1-\mu)\left(q_{t-1} + \mu q_{t-2} + \mu^2 q_{t-3} + ...\right) && (16) \\ q_t &= (1-\mu)\left(\pi^e_{t+1,t} + \mu\pi^e_{t+2,t} + \mu^2\pi^e_{t+3,t} + ...\right) && (17) \end{aligned}$$

where c_t denotes core inflation, q_t denotes contract inflation and $\pi^e_{t+s,t}$ denotes expectations at t of inflation at time $t+s$. We immediately observe that setting $\mu = 0$, we obtain the models analyzed in section 3. They are thus a special case of this model.

Currie and Levine solve the model, numerically in the case of commitment and (partly) analytically under discretion, but their paper analyzes the welfare loss under commitment, discretion and under a simple rule. Their results will therefore not be of much value for my analysis, but I will benefit very much from their solution methods.

To solve the model, we need to reformulate it. First, note that (16) implies the following dynamics of c:

$$c_{t+1} = \mu c_t + (1-\mu) q_t.$$

Let us then define

$$\tilde{q}_t = \pi_t + \mu\pi^e_{t+1,t} + \mu^2\pi^e_{t+2,t} + ...$$

We then get

$$\tilde{q}^e_{t+1,t} = \mu^{-1}(\tilde{q}_t - \pi_t).$$

Now, after defining $\delta = \mu^{-1}(1-\mu)^2$, the dynamics of the model can be summarized by

$$\begin{bmatrix} \pi^* \\ y^* \\ c_{t+1} \\ \epsilon_{t+1} \\ \tilde{q}^e_{t+1,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu^{-1} \end{bmatrix} \begin{bmatrix} \pi^* \\ y^* \\ c_t \\ \epsilon_t \\ \tilde{q}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\delta \\ 0 \\ -\mu^{-1} \end{bmatrix} \pi_t + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \epsilon_{t+1} \\ 0 \end{bmatrix}$$

or in condensed form

$$Z_{t+1} = AZ_t + B\pi_t + \xi_{t+1}. \qquad (18)$$

The state vector is partitioned according to the predetermined and non-predetermined variables as $Z_t = \begin{bmatrix} X_t & \tilde{q}^e_t \end{bmatrix}'$ and the vector of innovations is partitioned as $\xi_{t+1} = \begin{bmatrix} \xi_{x,t+1} & 0 \end{bmatrix}'$.

We also need to write the objective function in terms of the variables in the model. Let us define the target variables

$$\begin{bmatrix} y_t - y^* \\ \pi_t - \pi^* \end{bmatrix} = \begin{bmatrix} 0 & -1 & -\alpha & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} Z_t + \begin{bmatrix} \alpha \\ 1 \end{bmatrix} \pi_t,$$

which I express in condensed form as

$$s_t = GZ_t + H\pi_t.$$

Now, the loss function (1) can be written in terms of the target variables

$$L_t = \frac{1}{2}s_t'Q_1s_t,$$

where

$$Q_1 = \begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix}.$$

This enables us to express the loss function as a quadratic form of the state and control variables only

$$\begin{aligned} s_t'Q_1s_t &= (GZ_t + H\pi_t)'\,Q_1\,(GZ_t + H\pi_t) \\ &= Z_t'G'Q_1GZ_t + \pi_t'H'Q_1GZ_t + Z_t'G'Q_1H\pi_t + \pi_t'H'Q_1H\pi_t \\ &= Z_t'QZ_t + Z_t'U\pi_t + \pi_t'U'Z_t + \pi_t'R\pi_t \end{aligned} \tag{19}$$

where $Q = G'Q_1G,\ U = G'Q_1H$ and $R = H'Q_1H.$

5.1. Commitment

The algorithm for solving the model numerically under commitment is discussed in appendix A. The optimal policy rule turns out to be dynamically equivalent to that of the model with a traditional Phillips curve in the sense that simulated paths for inflation and output are exactly the same in the two economies if they are hit by the same sequence of shocks. This result follows from the fact that contracting is purely forward looking. A shock to output at time t will affect contemporaneous inflation but not successive inflation. From equation (17) it is then evident that contracts at time t will not be altered either. From the beginning it was not obvious that shocks would have effects on contemporaneous inflation only, and thus neither that contracting would be static.

5.2. Discretion

As noted by Currie & Levine (1993), an analytical solution to the model is easily obtained under discretion. Expectations on the central bank's behavior are exogenously given and will not be altered whatever the bank does. The central bank will therefore minimize the period loss function (1) at each time period, knowing that the same minimization will be carried out next period and that agents expect this to be the case. The problem is thus

$$\min_{\pi_t} \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda\,(y_t - y^*)^2\right]$$

subject to (15) - (17). The first order condition yields

$$\pi_t - \pi^* + \lambda\alpha\left[\alpha\,(\pi_t - c_t) - y^* + \epsilon_t\right],$$

implying

$$\pi_t = \left(1 + \lambda\alpha^2\right)^{-1} \left(\pi^* + \lambda\alpha^2 c_t + \lambda\alpha y^* - \lambda\alpha\epsilon_t\right). \tag{20}$$

Now, since both contract inflation, q_t, and core inflation, c_t, only depend on expectations and expectations are given, these variables will be constant[9]. We then observe that

$$\pi^e_{t+s,t} = ... = \pi^e_{t+2,t} = \pi^e_{t+1,t} \Rightarrow c_t = \pi^e. \tag{21}$$

Taking expectations of (20) and using (21) yields

$$\pi^e = \left(1 + \lambda\alpha^2\right)^{-1} \left(\pi^* + \lambda\alpha^2 \pi^e + \lambda\alpha y^*\right),$$

which implies that

$$c_t = \pi^e = \pi^* + \lambda a y^*.$$

We substitute for c_t in (20) to obtain the policy rule

$$\begin{aligned} \pi_t &= \left(1 + \lambda\alpha^2\right)^{-1} \left[\pi^* + \lambda\alpha^2 \left(\pi^* + \lambda a y^*\right) + \lambda\alpha y^* - \lambda\alpha\epsilon_t\right] \\ &= \pi^* + \lambda\alpha y^* - \frac{\lambda\alpha}{1 + \lambda\alpha^2}\epsilon_t. \end{aligned} \tag{22}$$

A comparison of (22) with the policy rule under discretion in the traditional model, equation (7), shows that the two policy rules are identical.

5.3. Properties of inflation and output

Just as in the previous section, we have seen that the policy rules, both under commitment and under discretion, imply that inflation and output follow the same paths as in the traditional model. The properties of inflation and output are thus unchanged. However, I have not shown that the problems solved in this model and in the traditional model are equivalent. It is therefore not immediately apparent that the same linear inflation contract still yields optimality. As expected though, a short computation is enough to show that the same linear inflation contract, $k_0 = \lambda\alpha y^*$, renders optimality in this model also.

6. Long term nominal contracting and persistence

So far, I have abstracted from output persistence in the analysis. Given that we have solved the model in the previous section, it is easy to allow for persistence. The economy is then described by

$$y_t = \rho y_{t-1} + \alpha\left(\pi_t - c_t\right) + \epsilon_t$$

and equations (16) and (17).

[9]This argument comes from Currie & Levine (1993).

We now augment the state vector with lagged output, y_{t-1}, and obtain

$$\begin{bmatrix} \pi^* \\ y^* \\ c_{t+1} \\ \epsilon_{t+1} \\ y_t \\ \tilde{q}^e_{t+1,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & 1 & \rho & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu^{-1} \end{bmatrix} \begin{bmatrix} \pi^* \\ y^* \\ c_t \\ \epsilon_t \\ y_{t-1} \\ \tilde{q}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\delta \\ 0 \\ \alpha \\ -\mu^{-1} \end{bmatrix} \pi_t + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \epsilon_{t+1} \\ 0 \\ 0 \end{bmatrix}.$$

The target variables are given by

$$\begin{bmatrix} y_t - y^* \\ \pi_t - \pi^* \end{bmatrix} = \begin{bmatrix} 0 & -1 & -\alpha & 1 & \rho & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} Z_t + \begin{bmatrix} \alpha \\ 1 \end{bmatrix} \pi_t.$$

6.1. Commitment

Simulations now show that there is a difference between the traditional model and the model introduced in this section (henceforth referred to as the Buiter-Miller model). The difference under commitment is small, though. As can be seen in figure 1 and figure 2, the paths of inflation are very similar in the two models when they are hit by the same sequence of shocks. The magnitude of the responses are equal, but shocks have more long-lasting effects when contracts have longer duration, as in the Buiter-Miller model.

The paths of output are also very similar, as is apparent from figure 3 and figure 4, but output seems to return to the natural level slightly faster in the Buiter-Miller model than in the traditional model.

In the standard model, only contemporaneous inflation responses to a shock will have effects on output. If there were later inflation responses, these would be rationally foreseen and therefore not have any effects on output. A shock at time t will thus only affect contemporaneous inflation, but due to persistence, output will be affected also in successive periods. In the Buiter-Miller model, on the other hand, prices and wages are set for more than one period. Although any inflation responses in period $t+s$ to a shock at t will be foreseen already in period t, agents cannot change their contracts. There is therefore a scope for durable inflation responses in the Buiter-Miller model. Sticky prices and wages increase the possibility of using inflation to stabilize output. The central bank will use some of this increased freedom to let not only output, but also inflation bear the effects of the shock.

In table 1, simulated volatilities of inflation and output are reported for different parameter values. As expected, the volatility of output is slightly lower in the Buiter-Miller model than in the traditional model, but it is not a general result that inflation volatility is higher. When the central bank's weight on output stability is high and when contracts have long duration, inflation volatility tends to be lower in the Buiter-Miller model. The reason for the link between a high weight on output stability and gains in inflation stability is not obvious, but remember that we only look at relative volatilities and that the increased weight on output stability applies for the standard model as well. The best explanation I can come up with is that since inflation is a more powerful instrument in the Buiter-Miller model, it will be used to some extent even if inflation stability has high priority.

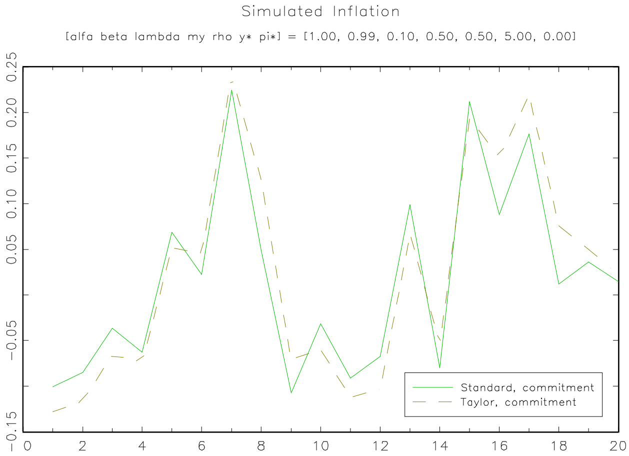

Figure 1: Simulated path of inflation.

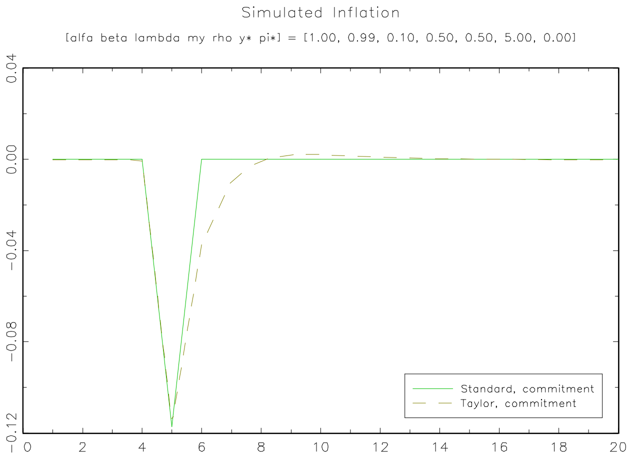

Figure 2: Impulse-response of inflation to a positive output shock in period 5.

Figure 3: Simulated path of output.

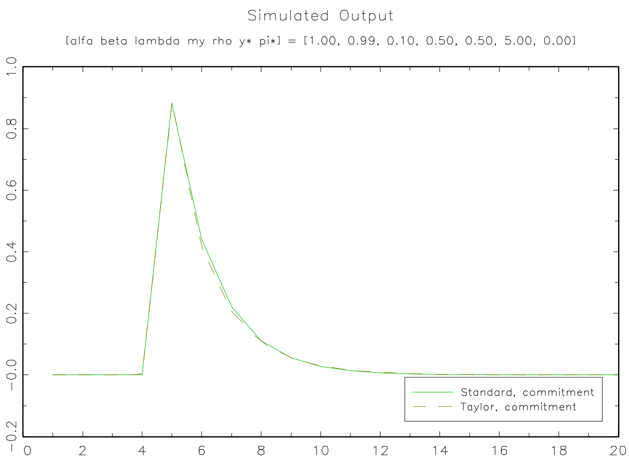

Figure 4: Impulse-response of output to a positive output shock in period 5.

The general intuition to these results is that the central bank has a more powerful toolbox when contracts are set for several periods. Under commitment, this toolbox will be used in an optimal way and it will enable the central bank to better control output volatility, sometimes without increasing inflation volatility.

6.2. Discretion

When the central bank acts under discretion, simulations of the model still indicate that the difference between the models is small. Figure 5 and figure 6 show that the behavior of inflation is almost identical in the two models, but that the inflation bias is smaller in the Buiter-Miller model than in the standard model. Simulations show that this result is amplified when either contract length, the degree of persistence or the effect of inflation surprises on output is increased, i.e. when the parameters μ, ρ or α are increased.

The intuition for this result is not clear to me. Obviously, the benefits of surprise inflation to the central bank will be greater when contracts are fixed for several periods, since the surprise inflation will make inflation higher than contracted and expected inflation not only today but also in the future. What is important, though, is the marginal benefit of surprise inflation. The marginal cost is linear,

$$MC = \pi_t - \pi^*$$

and the same in both models. Some calculations on a simplified model indicate that the marginal benefit will decrease more rapidly and that marginal benefit will be lower for given inflation expectations in the Buiter-Miller model than in the traditional model[10]. I think the mechanism behind this is that a certain inflation surprise has larger effects in the Buiter-Miller model. Since increases in output are most appreciated when output is far from the desired level, the large response to the inflation surprise will mean that output moves rapidly towards levels where inflation surprises are less valuable.

It still remains true that output follows almost exactly the same path as it does in the traditional model, which can be seen in figure 7 and figure 8. Moreover, in table 1 we see that inflation always is less volatile in the Buiter-Miller model than in the traditional model. Output is less volatile if the central bank's weight on output stability, λ, is small which is a surprising result. This result is similar to the result obtained under commitment earlier. Again, it is hard to explain the result, but my argument is the same as in the previous subsection.

[10] See appendix B.

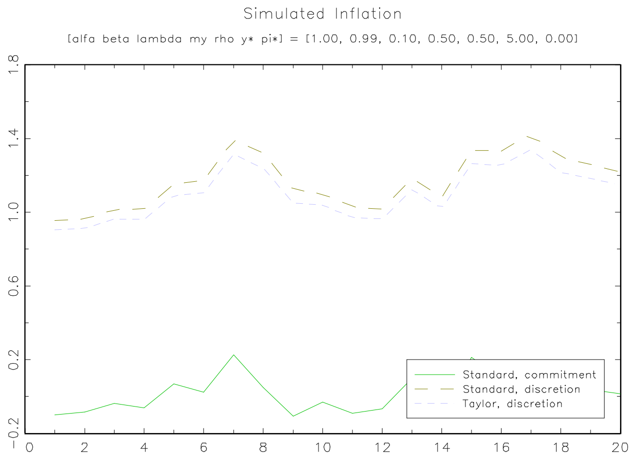

Figure 5: Simulated path of inflation.

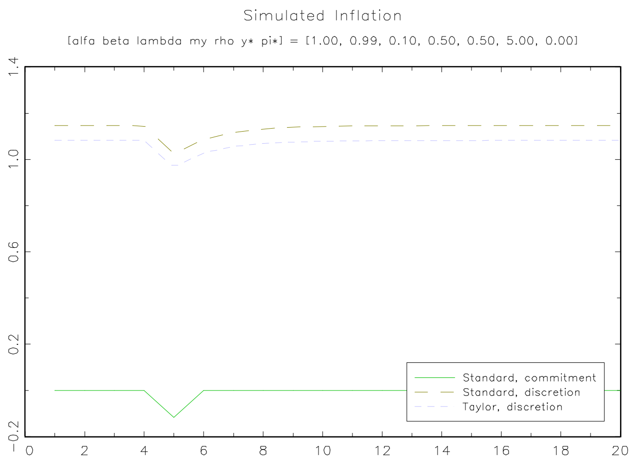

Figure 6: Impulse-response of inflation to a positive output shock in period 5.

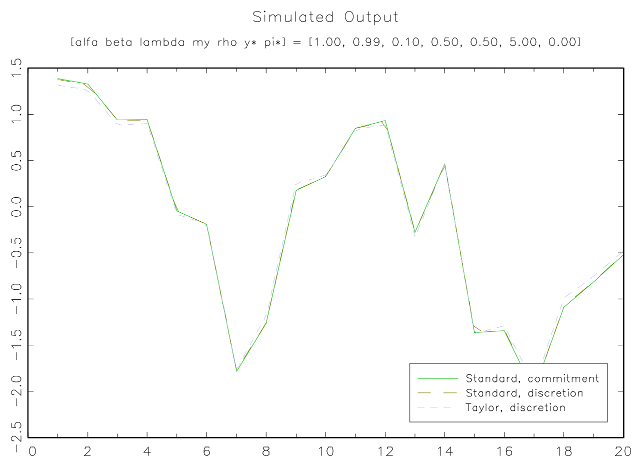

Figure 7: Simulated path of output.

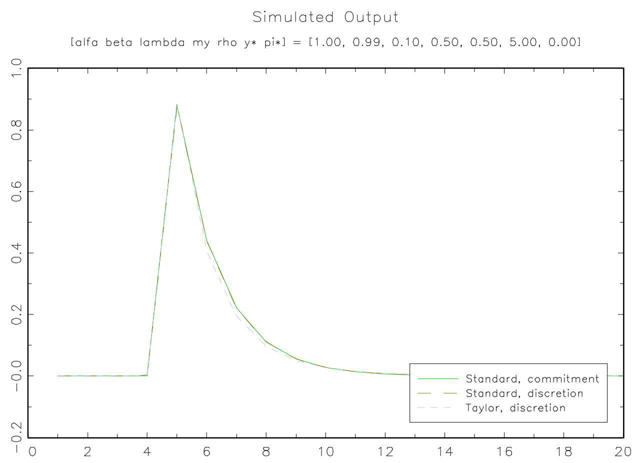

Figure 8: Impulse-response of output to a positive output shock in period 5.

Table 1: Simulated Volatility of Inflation and Output

			Std. Deviation of Inflation				Std. Deviation of Output			
			Traditional		Buiter-Miller		Traditional		Buiter-Miller	
μ	ρ	λ	Com.	Dis.	Com.	Dis.	Com.	Dis.	Com.	Dis.
.75	.25	.10	.0973	.1013	.0993	.0996	.9444	.9438	.9399	.9396
.50	.50	.10	.1161	.1399	.1207	.1312	1.0101	1.0044	.9969	.9884
.75	.50	.10	.1183	.1410	.1254	.1286	1.0183	1.0126	.9951	.9918
.10	.50	.50	.4078	.5894	.4085	.5399	.7151	.6009	.7135	.6274
.10	.25	1.00	.5203	.5594	.5209	.5456	.5054	.4848	.5048	.4898
.25	.25	1.00	.5213	.5594	.5211	.5336	.5055	.4849	.5035	.4951
.50	.25	1.00	.5203	.5563	.5178	.5206	.5027	.4822	.4996	.4971
.75	.25	1.00	.5168	.5527	.5125	.5130	.4994	.4790	.4962	.4958
.25	.25	2.00	.6865	.7518	.6854	.6988	.3336	.2929	.3324	.3233

Note: Parameter values used: $\alpha = 1$, $\beta = 0.99$, $\pi^* = 0$ and $y^* = 5$. Simulations performed over 5000 periods. Com. = commitment, Dis. = discretion.

6.3. Properties of inflation and output

As has been apparent earlier in this section, the introduction of persistence in output made our models behave differently. The simulations I have performed with different parameter values in the allowed range suggest that the difference between the two models is small[11]. There is still an inflation bias but it tends to be lower in the Buiter-Miller model than in the traditional model, in particular when contracts have long duration, when output persistence is severe and when inflation surprises have strong effects on output.

From table 1, we see that the simulated standard deviation of inflation always is higher under discretion than under commitment, and that the opposite holds for output. The simulations thus strongly indicate that inflation is more volatile and output more stable under discretion than under commitment, just as in the traditional model.

In the traditional model, optimality will be achieved under discretion if the optimal state-dependent linear contract

$$k_0 = \frac{\lambda \alpha y^*}{1-\beta\rho} \text{ and } k_1 = -\frac{\lambda\alpha\rho}{1-\beta\rho^2} \tag{23}$$

is implemented. To examine the effects of implementing a linear inflation contract in the Buiter-Miller model, I altered the loss function and performed new simulations. The target variables are now

$$\begin{bmatrix} y_t - y^* \\ \pi_t - \pi^* \\ k_0 + k_1 y_{t-1} \end{bmatrix} = \begin{bmatrix} 0 & -1 & -\alpha & 1 & \rho & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{k_0}{y^*} & 0 & 0 & k_1 & 0 \end{bmatrix} Z_t + \begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix} \pi_t$$

[11] I have mainly laborated with contract length, μ, the degree of persistence, ρ, and with the central bank's weight on output stability, λ. When ρ and/or λ are too high, the numerical algorithm does not converge. The same existence conditions as in the traditional model seem to guarantee that a solution can be found. (The existence conditions are reported in Svensson [1995]).

and the matrix Q_1 is given by

$$Q_1 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

The natural first attempt was to use the parameter values which are optimal in the traditional model, i.e. those given by (23). Somewhat surprisingly, considering that the inflation biases are not the same in the two models, this contract turned out to yield optimality also in the Buiter-Miller model.

7. Concluding remarks

In this paper, I have examined how sensitive the results in the literature on time consistent monetary policy are to the specific modeling of the supply side of the economy. In particular, I have been interested in three qualitative properties of the economy under discretionary monetary policy, namely the inflation bias, the stabilization bias when there is output persistence and the possibility to obtain optimality with state-contingent linear inflation contracts. On the way, I have also found some interesting quantitative results.

The analysis strongly indicates that the results obtained in the previous literature are very robust to changes in the specification of the supply side. First, as noted by Roberts (1995), a variety of different models of the supply side end up with identical Phillips curves.

Second, the different Phillips curves I have analyzed yield either the same or very similar results as when the expectations augmented Phillips curve is used. If there is no persistence in output *or* if contracts are written for only one period ahead, all models I have examined yield exactly the same results as when the traditional, expectations augmented Phillips curve is used.

If output persistence and long-lasting contracts are introduced, inflation and output will behave slightly differently than when the traditional Phillips curve is used. The difference appears to be quantitatively small though, and all the qualitative results still hold – there is still both an inflation bias and a stabilization bias under discretion and the same state-contingent linear inflation contract as in the traditional model makes the central bank behave as under commitment and thus renders optimality.

This last result is the one I find most interesting and important. Persson and Tabellini (1993) showed that a linear inflation contract will give optimality for a broad class of loss functions. My exercise indicates that this is true also for a broad class of models of the supply side. The optimal contract will depend on some parameter values which are hard to identify and also on the shape of the loss function. Anyhow, these findings support the view that the central bank shall be punished, explicitly or implicitly, when inflation becomes higher than the inflation target. As noted by other authors, the implementation and announcement of an inflation target might impose such an implicit contract in itself.

Appendices

A. Numerical solution method

The method for solving the system

$$\min_{\{\pi_t\}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(Z_t' Q Z_t + Z_t' U \pi_t + \pi_t' U' Z_t + \pi_t' R \pi_t \right) \tag{24}$$

subject to (18) is described in Backus & Driffill (1986). Note that this is not a standard quadratic-linear dynamic optimal control problem, since the state vector contains the forward looking variable $\tilde{q}^e$. However, Backus and Driffill show that the solution method is similar to that of the standard optimal control problem. The method is also described in Svensson (1994) and in chapter 6 of Currie & Levine (1993).

A.1. Commitment

We assume agents have rational expectations, implying $\tilde{q}_{t+1} = \tilde{q}^e_{t+1,t} + \nu_{t+1}$, where the forecast error ν_{t+1} is uncorrelated with all information available at time t. The Bellman equation is then the traditional

$$J(Z_t) = \frac{1}{2} Z_t' V Z_t = \min_{\pi_t} \frac{1}{2} \left(Z_t' Q Z_t + Z_t' U \pi_t + \pi_t' U' Z_t + \pi_t' R \pi_t + \beta E_t Z_{t+1}' V Z_{t+1}' \right)$$

subject to (18). We substitute from the transition equation in to the objective and obtains

$$\begin{aligned} J(Z_t) &= \frac{1}{2} Z_t' V Z_t = \min_{\pi_t} \frac{1}{2} \{ Z_t' Q Z_t + Z_t' U \pi_t + \pi_t' U' Z_t + \pi_t' R \pi_t + & (25) \\ & + \beta E_t \left(A Z_t + B \pi_t + \bar{\epsilon}_{t+1} \right)' V \left(A Z_t + B \pi_t + \bar{\epsilon}_{t+1} \right) \Big\} . & (26) \end{aligned}$$

Here $\bar{\epsilon}$ is defined to be the vector of exogenous shocks, including ν. The first order condition is then given by

$$U' Z_t + R \pi_t + \beta B' V A Z_t + \beta B' V B \pi_t = 0,$$

which implies that

$$\begin{aligned} \pi_t &= - \left(R + \beta B' V B \right)^{-1} \left(U' + \beta B' V A \right) Z_t \\ &= -F Z_t. \end{aligned} \tag{27}$$

Now, substituting (27) into (26) yields

$$\begin{aligned} Z_t' V Z_t &= Z_t' Q Z_t - Z_t' U F Z_t - Z_t' F' U' Z_t + Z_t' F' R F Z_t + \\ & + Z_t' \beta \left(A - BF \right)' V \left(A - BF \right) Z_t + k, \end{aligned}$$

where k is a constant. Disregarding the constant term, this implies that

$$V = Q - UF - F'U' + F'RF + \beta \left(A - BF \right)' V \left(A - BF \right). \tag{28}$$

The value of V is calculated numerically by using the expression for F in (27) in equation (28) and iterating on (28), starting with some arbitrary positive semidefinite matrix V, e.g. $V_0 = I$.

Now, we have found the decision rule and the optimal value function. One problem remains to be solved though, namely that of finding the initial value of $\tilde{q}$. We need that value to get the initial value of $Z_0 = \begin{bmatrix} X_0 & \tilde{q}_0 \end{bmatrix}'$, where we already know X_0. I use the Lagrangian method (see Backus & Driffill, section 2.2) to calculate shadow prices for $\tilde{q}$. The Lagrangian to (24) subject to (18) is given by

$$\begin{aligned}
&\pounds_0 (Z_0, Z_1, ..., \pi_0, \pi_1, ..., \mu_1, \mu_2, ...) \\
= \; & \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(Z_t' Q Z_t + Z_t' U \pi_t + \pi_t' U' Z_t + \pi_t' R \pi_t \right) + \\
&+ \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mu_{t+1} \left(A Z_t + B \pi_t + \bar{\epsilon}_{t+1} - Z_{t+1} \right)
\end{aligned}$$

where μ_{t+1} is the present value Lagrange multiplicator associated with the restriction (18) at time t. Let us partition μ according to the predetermined and non-predetermined variables of the state vector

$$\mu_t = \begin{bmatrix} \mu_{X,t} \\ \mu_{\tilde{q},t} \end{bmatrix}.$$

From the definition of shadow prices we know that

$$\mu_{t+1} = \beta^t \frac{\partial J(Z_t)}{\partial Z_t} = \beta^t V Z_t.$$

Partitioning V according to the predetermined and non-predetermined variables and using the partitionings of Z_t and μ_t previously specified, we can write this as

$$\begin{bmatrix} \mu_{X,t+1} \\ \mu_{\tilde{q},t+1} \end{bmatrix} = \beta^t \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \begin{bmatrix} X_t \\ \tilde{q}_t \end{bmatrix}. \qquad (29)$$

This implies that

$$\tilde{q}_t = \begin{bmatrix} -V_{22}^{-1} V_{21} & V_{22}^{-1} \end{bmatrix} \begin{bmatrix} X_t \\ p_{\tilde{q},t} \end{bmatrix} \equiv L \begin{bmatrix} X_t \\ p_{\tilde{q},t} \end{bmatrix} \qquad (30)$$

where I have defined the current value shadow prices $p_t = \beta^{-t} \mu_{\tilde{q},t+1}$. We now get an expression for the forecast error,

$$v_t = \tilde{q}_t - \tilde{q}_t^e = -V_{22}^{-1} V_{21} \xi_t,$$

since $X_t - E_{t-1} X_t = \xi_t$. This gives us the path for $\tilde{q}$.

We know that $p_{\tilde{q},0} = 0$ since that is the shadow price for the *ex ante* optimization problem, and the policy rule can be chosen with out restrictions. This gives us an initial value of $\tilde{q}$.

The solution to the problem can now be summarized as

$$\pi_t = -F Z_t$$

$$\begin{bmatrix} X_{t+1} \\ \tilde{q}_{t+1} \end{bmatrix} = (A - BF) \begin{bmatrix} X_t \\ \tilde{q}_t \end{bmatrix} + \begin{bmatrix} I \\ -V_{22}^{-1} V_{21} \end{bmatrix} \begin{bmatrix} \xi_{X,t+1} \\ 0 \end{bmatrix},$$

$$X_0 \text{ given and } \tilde{q}_0 = -V_{22}^{-1} V_{21} X_0.$$

A.2. Discretion

The algorithm used to solve the model under discretion is described in Svensson (1994). The algorithm is:

1. Partition the matrices A, B, Q, and U as:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \text{ and } U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}.$$

2. Initiate $V^* = 0_{m\times m}$ and $C = 0_{1\times n}$, where m is the number of non-predetermined variables and n is the number of predetermined variables in the system.

3. Then calculate

$$\begin{aligned}
D &= (A_{22} - CA_{12})^{-1}(CA_{11} - A_{21}) \\
G &= (A_{22} - CA_{12})^{-1}(CB_1 - B_2) \\
A^* &= A_{11} + A_{12}D \\
B^* &= A_{12}G + B_1 \\
Q^* &= Q_{11} + Q_{12}D + D' * Q_{21} + D'Q_{22}D \\
U^* &= Q_{12}G + D'Q_{22}G + U_1 + D'U_2 \\
R^* &= R + G'Q_{22}G + G'U_2G \\
F^* &= (R^* - \beta B^{*\prime}V^*B^*)^{-1}(U^{*\prime} + \beta B^{*\prime}V^*A^*) \\
V^* &= Q^* - U^*F^* - F^{*\prime}U^{*\prime} + F^{*\prime}R^*F^* + \beta(A^* - B^*F^*)'V^*(A^* - B^*F^*) \\
C &= D - GF^*
\end{aligned}$$

   and iterate on these calculations until the system has converged.

4. The laws of motion for the system is then given by

$$\begin{aligned}
Z_{t+1} &= M^*Z_t + \bar{\varepsilon}_{t+1} \\
&\equiv \begin{bmatrix} A^* - B^*F^* & 0 \\ C(A^* - B^*F^*) & 0 \end{bmatrix} Z_t + \begin{bmatrix} I \\ C \end{bmatrix} \xi_{X,t+1}.
\end{aligned}$$

B. The inflation bias

In this appendix, I try to clarify why the inflation bias becomes lower in the Buiter-Miller model than in the standard model. The marginal cost of additional inflation is linear and the same in both models:

$$MC = \pi_t - \pi^*$$

As mentioned in the text, calculations on a simplified model indicate that the marginal benefit will decrease more rapidly and that marginal benefit will be lower for given inflation expectations in the Buiter-Miller model than in the traditional model.

Let us assume we are in a stable state where output is at its natural level, there are no exogenous shocks and where inflation expectations and contracts are at the same level

$$\pi^e = c_t = \pi^e_{t,t-1} = \pi^e_{t+s,t-1}.$$

Now, assume the central bank raises inflation unexpectedly and marginally to a new level, which is maintained also in period $t+1$, whereafter the inflation rate returns to the original level. Let us also assume that this future path of inflation is recognized by the agents. In the standard model, the marginal benefit of this action is

$$\begin{aligned} MB^s &= -\sum_{s=0}^{\infty} \beta^s \lambda \left(y_{t+s} - y^*\right) \frac{\partial y_{t+s}}{\partial \pi_t} \\ &= -\sum_{s=0}^{\infty} \beta^s \rho^s \lambda \alpha \left[\alpha \rho^s \left(\pi_t - \pi^e\right) - y^*\right] \\ &= \frac{\lambda \alpha}{1-\beta\rho} y^* - \frac{\lambda \alpha^2}{1-\beta\rho^2} \left(\pi_t - \pi^e\right). \end{aligned}$$

In the Buiter-Miller model, the surprise inflation will to some extent be a surprise also in $t+1$. Some calculations then yield

$$\begin{aligned} c_{t+1} &= c_t + \left(1 - 2\mu + \mu^2\right) \left(\pi_{t+1} - c_t\right) \\ &= c_t + \left(1 - 2\mu + \mu^2\right) \Delta\pi \end{aligned}$$

$$\Longrightarrow$$

$$\pi_{t+1} - c_{t+1} = \mu \left(2 - \mu\right) \left(\pi_{t+1} - c_t\right).$$

The marginal benefit becomes

$$\begin{aligned} MB^{BM} &= -\sum_{s=0}^{\infty} \beta^s \lambda \left(y_{t+s} - y^*\right) \frac{\partial y_{t+s}}{\partial \pi_t} \\ &= -\lambda \alpha \left[\alpha \left(\pi_t - c_t\right) - y^*\right] + \\ &\quad -\sum_{s=1}^{\infty} \beta^s \lambda \rho^{s-1} \left[\rho \alpha + \alpha \mu \left(2 - \mu\right)\right] \times \\ &\quad \left[\alpha \rho^s \left(\pi_t - c_t\right) + \alpha \rho^{s-1} \mu \left(2 - \mu\right) \left(\pi_{t+1} - c_t\right) - y^*\right] \\ &= -\sum_{s=0}^{\infty} \beta^s \rho^s \alpha \lambda \left[\alpha \rho^s \left(\pi_t - c_t\right) - y^*\right] + \\ &\quad -\sum_{s=1}^{\infty} \beta^s \rho^{s-1} \alpha \mu \left(2 - \mu\right) \lambda \left[\alpha \rho^s \left(\pi_t - c_t\right) - y^*\right] \\ &\quad -\sum_{s=1}^{\infty} \beta^s \rho^{2s-1} \alpha^2 \lambda \mu \left(2 - \mu\right) \left(\pi_{t+1} - c_t\right) \\ &\quad -\sum_{s=1}^{\infty} \beta^s \rho^{2s-2} \alpha^2 \mu^2 \left(2 - \mu\right)^2 \lambda \left(\pi_{t+1} - c_t\right) \end{aligned}$$

$$= \frac{\lambda\alpha}{1-\beta\rho}\left[1+\beta\mu\left(2-\mu\right)\right]y^* + \\ -\frac{\lambda\alpha^2}{1-\beta\rho^2}\left[1+2\beta\rho\mu\left(2-\mu\right)+\beta\mu^2\left(2-\mu\right)^2\right]\left(\pi_t - c_t\right)$$

The calculations result in marginal benefit functions

$$MB^S \approx a_1 y^* - a_2\left(\pi_t + \pi_t^e\right)$$

$$MB^{BM} \approx b_1 y^* - b_2\left(\pi_t + \pi_t^e\right)$$

for the standard model and the Buiter-Miller model respectively. We have seen that $b_1 > a_1$, $b_2 > a_2$ and that the relative difference between b_i and a_i is increasing in β, μ and ρ (not for $i = 1$), while the absolute difference is increasing in these parameters as well as in α and λ.

The equilibrium inflation bias is that which equates marginal cost with marginal benefits. A graphical analysis (see figure B.1) now shows that the inflation bias will be lower in the Buiter-Miller model, at least for some parameter values.

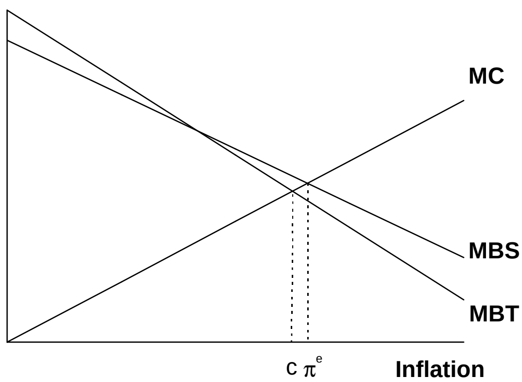

Figure B.1. Equilibrium inflation bias in the standard model (π^e) and in the Buiter-Miller model (c).

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